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Contaminant Spreading in Stratified Soils With Fractal Permeability Distribution

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Stochastic analysis of flow and transport in the subsurface usually assumes that the soil permeability is a stationary, homogeneous stochastic process with a finite variance. Some field data suggest, however, that the permeability distributions may have a fractal character with long-range correlations. It is of interest to investigate how the fractal character of permeability distribution influences the spreading and mixing process in porous media. The results of our analysis of this problem for perfectly stratified media with fractal distribution of permeability along the vertical are presented. Results were obtained for the transient and asymptotic dispersivities in the longitudinal direction. The results show that the macroscopic asymptotic dispersivity depends strongly on the fractal dimension of vertical permeability distribution. Specifically, the higher the fractal dimension, the lower the value of macroscopic dispersivity. The macroscopic dispersivity was found to be problem-scale dependent in transient (development) and asymptotic phases. Variance of fluctuation of concentration was also analyzed and found to be dependent on the fractal dimension. In this case, the higher fractal dimension results in more mixing of pore water and therefore smoother (smaller σ_x^2) concentration distribution.

INTRODUCTION

The impact of heterogeneities on flow and mass transport in groundwater has been investigated for about two decades. Usually, this type of investigation is performed using the stochastic, as opposed to the deterministic framework. This choice is not based on the assumption that the flow process itself is stochastic but rather on the recognition of the fact that the deterministic description of the parameter distributions would be impractical, if not impossible.

Initial research in this area did not consider the spatial structure of flow properties, assuming that either it behaved like the white noise process (lack of spatial correlation), or had the layered structure in the direction parallel or perpendicular to the flow (perfect correlation in one direction). The next step was to consider spatial correlation of flow properties. Various autocovariance functions, including anisotropic ones, were used to describe the spatial correlation [*Dagan*, 1984; *Gelhar and Axness*, 1983]. Excellent review of this research is given by *Gelhar* [1986]. However, in most of these efforts it was assumed that the correlation structure of parameter fluctuations is such that the fluctuation variance is bounded and reaches its asymptotic value when the volume of the analyzed region increases. The validity of this assumption for geologic formations has yet to be demonstrated. The field data from the rather homogeneous Borden site indicate that, at least for this site, the assumption is acceptable. Other sites, however, show scale-dependent variance and long-range correlations of subsurface properties [*Burrough*, 1981, 1983*a*, *b*; *Hewett*, 1986]. This evidence prompted us to investigate the statistical behavior of solute transport in heterogeneous systems whose properties exhibit long-range correlations. The statistics of such properties are described using the concept of fractal, self-similar objects. Following that, we examine the behavior of a relatively simple transport problem: two-dimensional (vertical cross section) solute transport in a perfectly stratified medium.

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FRACTAL PERMEABILITY DISTRIBUTION

Theoretically, fractal objects are characterized by their self-similar structure on all scales and thus have partial correlations over long ranges. Such self-similarity may be exact, as in the case of the Koch Snow Flake, or statistical, as in the case of natural objects [*Mandelbrot*, 1983]. The basic assumption of the classical stochastic analysis of subsurface properties is that the variance of increments, or variogram, is bounded by the property's variance [*Journel and Huijbregts*, 1989].

$$\lim_{l \to \text{infinity}} \gamma(l) = 0.5E\{[X(z+l) - X(z)]^2\} = \sigma_x^2 \quad (1)$$

where the $\gamma(l)$ is the variogram at l distance lag, $X(z)$ is the subsurface properties at position z, and σ_x^2 is the property's variance.

One such variogram, used frequently in mining exploration, is the spherical variogram

$$\gamma(l) = \sigma^2\left[\frac{3}{2}\left(\frac{l}{a}\right) - 0.5\left(\frac{l}{a}\right)^3\right];\ l \le a$$
$$\gamma(l) = \sigma^2;\ l > a \quad (2)$$

The interval a, where the variogram reaches its maximum value ($\gamma(a) = \sigma^2$), is called the range. For fractal distributions, the variogram is not bounded because of the correlations over all scales, and is described by a power law [*Hewett and Behrens*, 1988]

$$\gamma(l) = \gamma_0 l^{2H} \quad (3)$$

where γ_0 is the variogram value at $l = 1$ and H is the fractal co-dimension, which is equal to the difference between the Euclidean dimension in which the fractal distribution is described and the fractal dimension of this distribution D. Thus for the vertical distribution of hydraulic conductivity K the codimension is given by

$$H = 2 - D \quad (4)$$

since such distribution is described in the two-dimensional space $K - z$. The statistical self-similarity of distributions

whose variogram is given by (3) is indicated by the fact that variations over any scale rl may be expressed in terms of the variations over a scale l by

$$\gamma(rl) = \gamma(l)r^{2H} \tag{5}$$

where r is the scaling factor.

An important conclusion from this relationship is that for fractal distributions the variance at any scale can be defined by the variance estimated at any other scale. This also implies that for the fractal distribution the variance is scale-dependent. The variogram may be related to the two-point autocovariance function. The definition of the autocovariance function of process X is given by

$$C_x(l) = E\{X(z + l)X(z)\} - E^2\{X(z)\} \tag{6}$$

The relationship between the variogram and the autocovariance function is defined as follows:

$$\gamma_x(l) = E\{X(z)^2\} - E^2\{X(z)\} - C_x(l) \tag{7}$$

Using this relationship and the Wiener-Khintchine theorem [*Hewett*, 1986],

$$C_x(l) = 2\int_0^\infty S_x(f)\cos(fl)\,df \tag{8}$$

where $S_x(f)$ is the spectral density of $X(z)$, it can be demonstrated that the spectral density of the fractal objects will also have the power law form,

$$S_x(f) = S_0 f^{-\beta} \tag{9}$$

where S_0 is the spectral density at $f = 1$ and $\beta = 2H + 1$. For fractal processes, β is constrained between 1 and 3 [*Hough*, 1989; *Voss*, 1985].

Thus the fractal dimension of a given process may be estimated by plotting the variogram and the spectral density of the process on logarithmic coordinates and calculating the slopes. Both slopes ($2H = 4 - 2D$ and $\beta = 5 - 2D$) should result in the same fractal dimension D.

Several sets of hydraulic conductivity data have been analyzed by *Ababou and Gelhar* [1990] and *Kemblowski and Chang* [1992]. Their analyses indicate that the vertical distribution of hydraulic conductivity is characterized by a highly irregular character, with the fractal dimension varying between 1.825 and 2.0. In the next section we will explore the relationship between this fractal dimension and solute spreading.

ANALYSIS OF SPREADING IN STRATIFIED POROUS MEDIA

The specific scenario considered in this paper assumes that groundwater flows horizontally in a perfectly stratified aquifer. Thus in the horizontal direction the correlation scale of permeability is infinite, whereas in the vertical direction permeability is described as a stochastic process. Mass transport of a conservative tracer in such a situation consists of horizontal advection, and horizontal and vertical local (pore level) dispersion. This transport problem is described in detail by *Gelhar et al.* [1979], who also derived for this scenario a general relationship between the stochastic structure of hydraulic conductivity and the longitudinal macrodispersivity. Using their general results, we will investigate the behavior of macrodispersivity for the case when the vertical distribution of permeability is fractal. The relationship between asymptotic longitudinal macrodispersivity, A, and the spectral density function of the vertical distribution of hydraulic conductivity, S_{ff}, is given by

$$A = 2\int_0^{+\infty} \frac{S(f)}{\mathbf{K}^2}\frac{(1 - e^{-bt})}{\alpha_T f^2}\,df, \tag{10}$$

where $b = \alpha_T U f^2$, α_T is the local transverse dispersivity, U is the average pore water velocity, K is the mean hydraulic conductivity, and f is the wave number. The large time limit of macrodispersivity A is given by

$$A_\infty = 2\int_0^{+\infty} \frac{S(f)}{\mathbf{K}^2}\frac{df}{\alpha_T f^2} \tag{11}$$

At this point we can start considering the behavior of solute transport spreading in media with fractal hydraulic conductivity distribution. Such distributions are characterized by the spectrum:

$$S(f) = S_0|f|^{-\beta} \tag{12}$$

It is apparent that the spectrum is not bounded when the frequency approaches zero. This can be understood when one remembers that the low frequencies are associated with large distances and that as the lag approaches infinity, so does the variogram of a fractal process. In this paper we assume that the self-similar behavior of log K distribution has a lower frequency limit. We use the characteristic cutoff frequency f_0, which is assumed to be the lower limit of self-similarity, and the associated characteristic length scale, L_0

$$f_0 = \frac{2\pi}{L_0} \tag{13}$$

It is assumed that any frequencies smaller than f_0 do not contribute to the spectral density function. The characteristic length scale L_0 may be indirectly related to the contaminant plume dimension. However, it is emphasized that L_0 does not represent the characteristic plume dimension in the strict sense. To account for the final size of a contaminant plume, the power spectrum (12) would have to be modified to represent a finite-size process. We can now redefine the spectral density as follows:

$$S(f) = S_0|f|^{-\beta} \qquad f > f_0 \tag{14a}$$

$$S(f) = 0 \qquad \text{otherwise} \tag{14b}$$

It is worth noting that although in general the fractal processes are considered nonstationary [*Mandelbrot*, 1983], these processes become quasi-stationary when the self-similarity occurs only over a certain range, bounded by an upper characteristic length scale L_0. In this case the variance of the process is bounded, and the autocovariance function depends only on the spatial lag. This characteristic allows us to use the *Gelhar et al.* [1979] results to analyze the spreading process in soils with fractal permeability distribution. Substituting the fractal spectrum into (11) leads to the following relationship for asymptotic macrodispersivity:

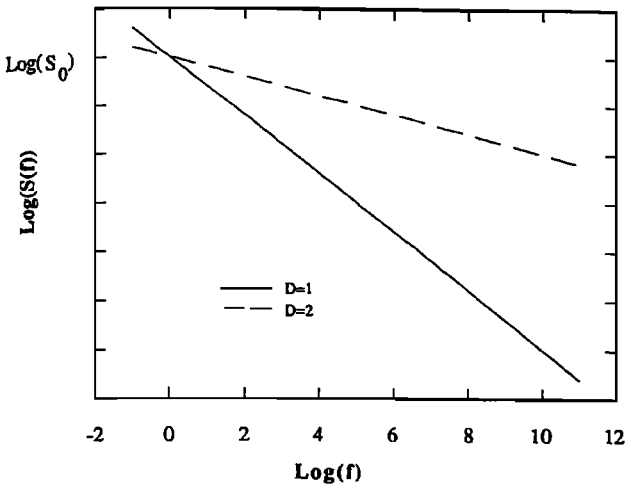

Fig. 1. Spectral density for $D = 1$ and $D = 2$, $S_0 = \text{const.}$

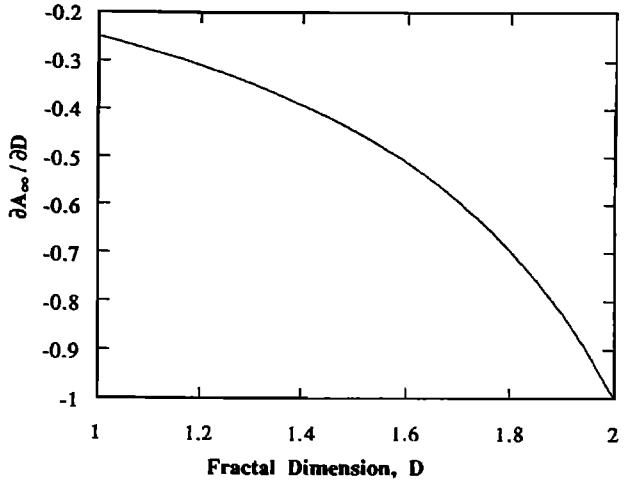

Fig. 2. $\partial A_\infty/\partial D$ for constant variance of K over L_0.

$$A_\infty = \frac{2S_0\left(\frac{L_0}{2\pi}\right)^{1+\beta}}{\alpha_T K^2(1+\beta)} = \frac{2S_0\left(\frac{L_0}{2\pi}\right)^{6-2D}}{\alpha_T K^2(6-2D)} \tag{15}$$

It is of interest to investigate the impact of fractal dimension D on macroscopic dispersivity. The derivative of A_∞ with respect to D is given by

$$\frac{\partial A_\infty}{\partial D} = -2A_\infty\left[\ln\left(\frac{L_0}{2\pi}\right) - \frac{1}{6-2D}\right] \tag{16}$$

Analysis of this relationship shows that, except for very small values of L_0, macroscopic dispersion, A_∞, decreases when the fractal dimension of K increases. This agrees with our physical intuition. As the fractal dimension increases, the front of the plume becomes "rougher," which leads to more mixing between the "layers." This enhanced vertical mixing reduces the horizontal spreading. Note that for the same reason macrodispersivity is inversely correlated with local transverse dispersivity. The positive correlation between macrodispersivity and the fractal dimension for small values of L_0 is an artifact which reflects the fact that by keeping S_0 constant and increasing D (which is equivalent to decreasing β), we increase the power (variance) of the process associated with higher frequencies (Figure 1).

A more appropriate way to analyze the correlation between the fractal dimension and macrodispersivity is to assume that the variance of K over L_0 remains constant. This variance can be estimated as follows for $D < 2$:

$$\sigma^2(L_0) = \sigma_0^2 = 2\int_{f_0}^{\infty} S(f)\,df = 2\int_{f_0}^{\infty} S_0 f^{-\beta}\,df = \frac{2S_0\left(\frac{L_0}{2\pi}\right)^{\beta-1}}{\beta-1} \tag{17}$$

Substituting this equation into (15) leads to

$$A_\infty = \frac{\sigma^2(L_0)\left(\frac{L_0}{2\pi}\right)^2(\beta-1)}{\alpha_T K^2(1+\beta)} = \frac{\sigma_0^2\left(\frac{L_0}{2\pi}\right)^2(4-2D)}{\alpha_T K^2(6-2D)} \tag{18}$$

The derivative of A_∞ with respect to D can be estimated as follows

$$\frac{\partial A_\infty}{\partial D} = \frac{2\sigma_0^2\left(\frac{L_0}{2\pi}\right)^2}{\alpha_T K^2(6-2D)}\left(\frac{4-2D}{6-2D} - 1\right) \tag{19}$$

The behavior of A_∞ for $\sigma_0^2(L_0/2\pi)^2/\alpha_T K^2 = 1$ is shown in Figure 2. It can be seen that $\partial A_\infty/\partial D$ is in this case (for constant variance σ_0^2) always negative, and its absolute magnitude increases with D. This again demonstrates that the higher fractal dimension enhances the vertical mixing process and therefore decreases the horizontal spreading.

Figure 3 depicts the behavior of the normalized asymptotic dispersivity (the dispersivity normalized with regard to the asymptotic dispersivity at $D = 1$) as a function of fractal

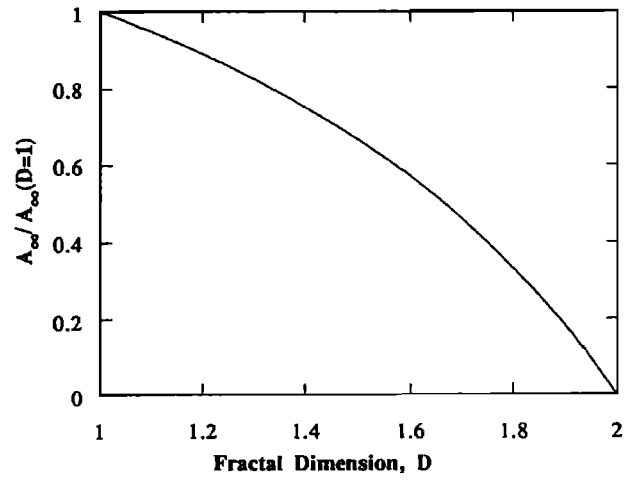

Fig. 3. Normalized asymptotic macrodispersivity as a function of fractal dimension of K.

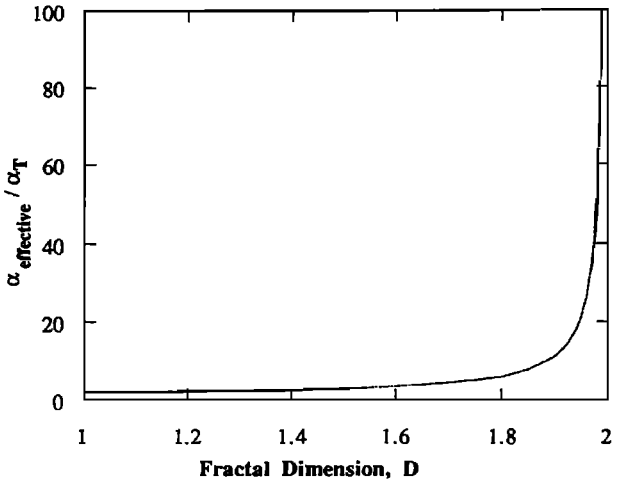

Fig. 4. Effective mixing coefficient as a function of D.

dimension D. It can be clearly seen that the asymptotic dispersivity decreases as the fractal dimension decreases. In fact, for the fractal dimension D approaching 2, we have the case of perfect mixing (which is similar to the case of α_T approaching infinity, although in this case the mixing is caused by the fact that the vertical distribution of K completely fills the x-z plane). Because of the perfect mixing effect there is no longitudinal spreading related to the heterogeneous velocity field. This problem can be illustrated by introducing the concept of an effective mixing coefficient, $\alpha_{\text{effective}}$, defined as follows:

$$\alpha_{\text{effective}} = \frac{\alpha_T(6 - 2D)}{(4 - 2D)} \tag{20}$$

Using this definition, the asymptotic macrodispersivity may be defined as

$$A_\infty = \frac{2\sigma_0^2\left(\frac{L_0}{2\pi}\right)^2}{\alpha_{\text{effective}}K^2} \tag{21}$$

The behavior of the normalized effective mixing coefficient, $\alpha_{\text{effective}}/\alpha_T$, as a function of the fractal dimension is shown in Figure 4. The effective mixing coefficient seems to be most sensitive to the changes in the fractal dimension in the region $1.8 < D < 2$. Field data suggest that the fractal dimension of the vertical distribution of hydraulic conductivity lies precisely in this region [*Ababou and Gelhar*, 1990]. It may be therefore difficult to estimate the effective mixing coefficient with adequate accuracy since this would require robust estimation procedures for D.

It is worthwhile noting that (18) is similar to the one obtained by *Gelhar et al.* [1979]. There are, however, two significant differences. First of all, in the fractal case, the variance of K is scale-dependent. Second, the correlation scale in this case is related to the characteristic scale, since the vertical hydraulic conductivity distribution has the correlation scale theoretically equal to infinity.

To better understand the behavior of the dispersion process, macrodispersivity is evaluated as a function of travel time, or more specifically, travel distance. Substituting (14) into (10) leads to the following result:

$$\frac{A}{A_\infty} = 1 - \exp\left(-\frac{\alpha_T x}{\left(\frac{L_0}{2\pi}\right)^2}\right) + \frac{\frac{\alpha_T x}{\left(\frac{L_0}{2\pi}\right)^2}\exp\left(-\frac{\alpha_T x}{\left(\frac{L_0}{2\pi}\right)^2}\right)}{2 - D} - \frac{\left(\frac{\alpha_T x}{\left(\frac{L_0}{2\pi}\right)^2}\right)^{3-D}\Gamma\left(D - 1, \frac{\alpha_T x}{\left(\frac{L_0}{2\pi}\right)^2}\right)}{2 - D} \tag{22}$$

where x is the average travel distance, $x = Ut$, and $\Gamma(\mu, \zeta)$ is the incomplete Gamma function. The transient behavior of macrodispersivity, as a function of normalized travel distance $X = \alpha_T x/(L_0/2\pi)^2$ and fractal dimension D, is shown in Figure 5. It appears that the asymptotic values of macrodispersivity is reached at X equal approximately 1 regardless of the fractal dimension magnitude. The fractal dimension influences the development of the spreading process only to a limited degree. Specifically, the asymptotic value is reached slightly faster for the higher fractal dimension of K. It is interesting to note that the travel distance required to reach the asymptotic behavior depends not only on the pore level transverse dispersivity but also on the scale of the problem, namely the characteristic length L_0. Thus the spreading process in fractal porous media appears to be scale-dependent in the development and asymptotic phases.

Fluctuations About the Mean Concentration Field

In addition to our interest in understanding the process of spreading of the mean concentration field, we are also interested in characterizing the concentration fluctuations about the mean concentration field. This interest is related to two problems. First of all, it is important to understand the uncertainties associated with using the mean concentration field to characterize contaminant transport. Related to this problem is the issue of field data interpretation and model calibration.

The second problem is more fundamental. For the last two

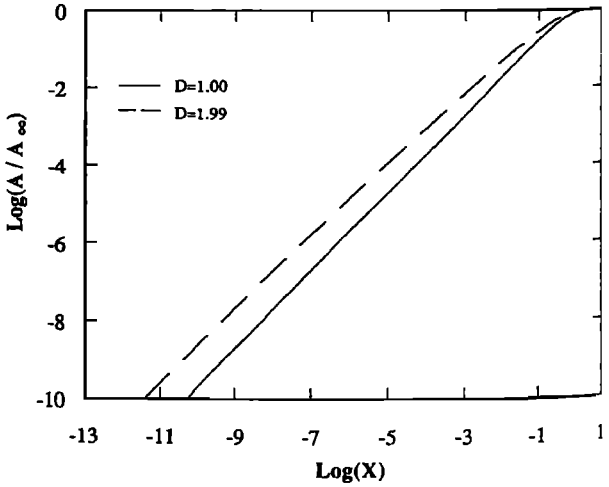

Fig. 5. Development of the spreading process.

decades, most of the groundwater literature concerned with contaminant transport has exhibited certain confusion between two processes: (1) spatial spreading of contaminant plumes, and (2) mixing of pore water. In fact, the very name used to describe the process of spreading-dispersion, is strongly suggestive of mixing. This confusion is perhaps not very important as long as we are analyzing transport of nonreactive tracers. However, as soon as we start looking at transport of reactive species, the difference between spreading and mixing becomes of utmost importance. One example of this problem is aerobic degradation of organics in groundwater. In order to facilitate biodegradation, oxygen and organics have to be actually mixed in the aqueous phase. Using the spreading coefficient to simulate the mixing process results in significant overestimation of biodegradation rate.

There are, in our perception, two principal questions related to this issue. These questions could be phrased as follows: (1) what does the macrodispersivity estimated for the average concentration field mean in the physical sense, and (2) how is this parameter related to the process of mixing between hydrocarbon and oxygen plumes. To simplify our discussion, we would like to refer to the results published by *Gelhar et al.* [1979]. We believe that this discussion could be readily extended to more complex situations. The asymptotic value of macrodispersivity obtained by *Gelhar et al.* [1979] for a specific autocorrelation function is given by

$$A_\infty = \frac{\sigma_k^2 \lambda^2}{3K^2 \alpha_T} \tag{23}$$

Consider the behavior of this parameter under the influence of two processes: (1) advective spreading, and (2) vertical mixing. First of all, it is obvious that for a lower magnitude of the vertical pore level dispersion (i.e., less mixing), the value of asymptotic dispersivity will be larger, resulting in larger horizontal spreading of the mean concentration field. Without mixing, the asymptotic dispersivity is equal to infinity, and its transient behavior is given by Mercado's results. Similarly, the mixing process is slowed down by larger vertical correlation scale of K (smaller local gradient of C). The spreading process is, however, enhanced by the variability (coefficient of variation) of the hydraulic conductivity field. A similar analysis could be performed using the results presented in this paper for fractal permeability distributions.

It is apparent that the less significant the mixing process is, the larger the apparent dispersion (spreading) will be. If this is the case, then obviously macrodispersivity (understood as a parameter that determines the smoothing of the mean concentration field) is not a good parameter to describe the mixing process. To get a better feel for these interwoven processes, one can look at the variance of the concentration field. If the concentration variance is zero, then the mean concentration is equal to the actual concentration, and the spreading and mixing processes can be described by the same parameter, macrodispersivity. As the concentration variance increases, the two processes drift apart. The variance of concentration fluctuations can be estimated by using the spectral density of the concentration fluctuations

$$\sigma_{cc}^2 = E[cc] = \int_{-\infty}^{\infty} S_{cc}\, df \tag{24}$$

The spectral density can be estimated using the following relationship:

$$S_{cc} dr = E(dZ_c dZ_c^*) \tag{25}$$

where [*Gelhar et al.*, 1979]:

$$dZ_c = U \frac{dZ_k}{K}\left[G\left(\frac{1-e^{-\alpha t}}{\alpha}\right) - \left(\frac{\partial G}{\partial t} - D_L \frac{\partial^2 G}{\partial \xi^2}\right) \cdot \left(\frac{1-(1+\alpha t)e^{-\alpha t}}{\alpha^2}\right)\right] \tag{26}$$

and dZ_c^* is the complex conjugate of dZ_c. Thus the spectrum of concentration fluctuations is

$$S_{cc} = \frac{U^2}{K^2}\left[G\left(\frac{1-e^{-\alpha t}}{\alpha}\right) - \left(\frac{\partial G}{\partial L} - D_L \frac{\partial^2 G}{\partial \xi^2}\right) \cdot \left(\frac{1-(1+\alpha t)e^{-\alpha t}}{\alpha^2}\right)\right]^2 S_{kk} \tag{27}$$

Using (27), we can estimate the variance of the concentration fluctuations

$$\sigma_{cc}^2 = E[cc] = \int_{-\infty}^{\infty} S_{cc}\, df = \frac{U^2}{K^2}\left[2G^2 \int_{f_0}^{\infty}\left(\frac{1-e^{-\alpha t}}{\alpha}\right)^2 \cdot S_{kk}\, df - 4G\left(\frac{\partial G}{\partial t} - D_L \frac{\partial^2 G}{\partial \xi^2}\right) \cdot \int_{f_0}^{\infty}\left(\frac{1-e^{-\alpha t}}{\alpha}\right)\left(\frac{1-(1+\alpha t)\, e^{-\alpha t}}{\alpha^2}\right) S_{kk}\, df + 2\left(\frac{\partial G}{\partial t} - D_L \frac{\partial^2 G}{\partial \xi^2}\right)^2 \cdot \int_{f_0}^{\infty}\left(\frac{1-(1+\alpha t)e^{-\alpha t}}{\alpha^2}\right)^2 S_{kk}\, df\right] \tag{28}$$

where

$$G = -\frac{\partial C}{\partial \xi} + \frac{3}{2}\frac{\partial^2 C}{\partial \xi^2} \tag{29}$$

$$\alpha = \alpha_T U f^2 \tag{30}$$

Equation (28) can be simplified by neglecting the higher-order derivatives of mean concentration (C) in the following terms:

$$G^2,\ G\left(\frac{\partial G}{\partial t} - D_L \frac{\partial^2 G}{\partial \xi^2}\right),\ \left(\frac{\partial G}{\partial t} - D_L \frac{\partial^2 G}{\partial \xi^2}\right)^2. \tag{31}$$

Using this simplification and the fractal spectrum of the permeability distribution, we can estimate the concentration variance

$$\sigma_{cc}^2 = \left(\frac{\partial C}{\partial \xi}\right)^2 \sigma_0^2 t^2 \left[\frac{2-D}{X^2(8-2D)} - \frac{\exp(-X)[2-\exp(-X)](2-D)}{X^2(8-2D)} + \frac{\exp(-X)[1-\exp(-X)](2-D)}{X(4-D)(3-D)} - \frac{\exp(-X)[1-2\exp(-X)]}{(4-D)(3-D)} + \frac{X^{2-D}}{(4-D)(3-D)}\Gamma(D-1, X) - \frac{2(2X)^{2-D}}{(4-D)(3-D)}\Gamma(D-1, 2X)\right] \quad (32)$$

Analysis of this result shows that the concentration variance decreases as the fractal dimension increases. This can be perhaps best demonstrated by studying the large-time behavior of concentration variance by assuming that time t approaches infinity. The result is given by

$$\lim_{t\to\infty, \text{ and } X\to\infty} \sigma_{cc}^2 = \frac{\left(\frac{\partial C}{\partial \xi}\right)^2 \sigma_0^2 \left(\frac{L_0}{2\pi}\right)^4 (2-D)}{(\alpha_T U)^2(8-2D)} \quad (33)$$

It is encouraging to see that the concentration variance does not grow at infinity. The asymptotic value of concentration variance depends strongly on the effective mixing coefficient, which in turn depends on the fractal dimension. To investigate this relationship, we analyze the behavior of a normalized concentration variance, which is here defined as follows:

$$\hat{\sigma}_{cc}^2|_{t\to\infty} = \frac{\sigma_{cc}^2(D)|_{t\to\infty}}{\sigma_{cc}^2(D=1)|_{t\to\infty}} = \frac{3(2-D)}{(4-D)} \quad (34)$$

The behavior of the normalized concentration variance is depicted in Figure 6. The asymptotic concentration variance decreases when the permeability distribution's fractal dimension increases. This behavior seems to be consistent with our results for macrodispersivity. The higher fractal dimension enhances the mixing process, which in turn smooths the concentration distribution (smaller concentration variance) and results in less horizontal spreading.

It is also of interest to investigate the transient behavior of the concentration variance. This can be performed by analyzing the behavior of a normalized concentration variance defined as follows:

$$\bar{\sigma}_{cc}^2(X, D) = \frac{\sigma_{cc}^2(X, D)}{\sigma_{cc}^2(X\to\infty, D=1)} = \left\{\frac{6(2-D)}{(8-2D)} - \frac{6\exp(-X)[2-\exp(-X)](2-D)}{(8-2D)} + \frac{6X\exp(-X)[1-\exp(-X)](2-D)}{(4-D)(3-D)} - \frac{6X^2\exp(-X)[1-2\exp(-X)]}{(4-D)(3-D)} + \frac{6X^{2-D}}{(4-D)(3-D)}\Gamma(D-1, X) - \frac{3(2X)^{4-D}}{(4-D)(3-D)}\Gamma(D-1, 2X)\right\} \quad (35)$$

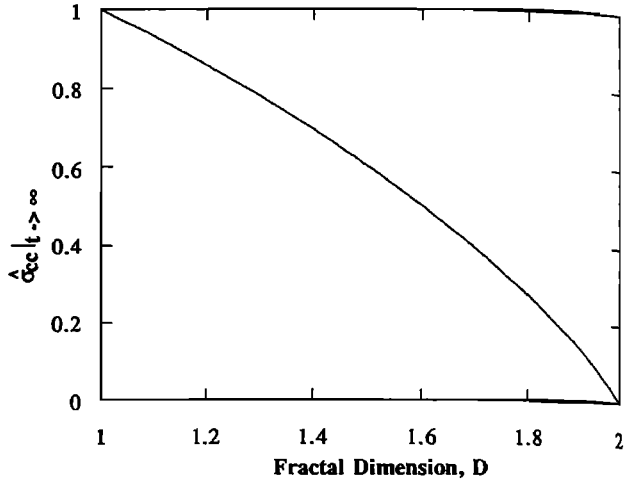

Fig. 6. Normalized asymptotic variance of concentration fluctuations.

Figure 7 depicts the behavior of this normalized concentration variance. The transient behavior of concentration variance seems to be similar for all values of the fractal dimension D. The concentration variance grows rapidly between $X = 0$ and $X = 1$. For $1 < X < 4$, the rate of growth is significantly smaller, and at about $X = 4$ the variance reaches its asymptotic value. It is worth noting that this behavior is quite similar to the transient development of macrodispersivity.

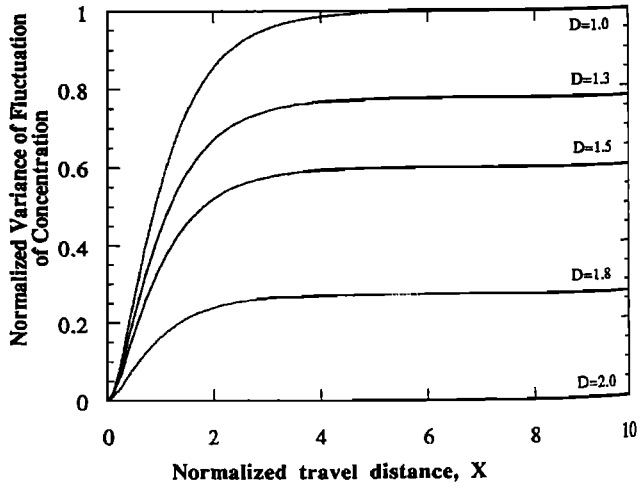

Fig. 7. Normalized variance of concentration fluctuation.

SUMMARY

The impact of the fractal dimension of the vertical distribution of hydraulic conductivity on the behavior of soluble plume spreading was investigated. It was found that the fractal dimension has a major impact on the value of asymptotic macrodispersivity. A higher fractal dimension enhances vertical mixing and results in less longitudinal advective spreading of the plume. For the fractal dimension approaching 2, the longitudinal spreading of the plume disappears altogether. Our results also indicate that the asymptotic dispersivity is a scale-dependent parameter. In particular, its value depends on the characteristic thickness of the soluble plume and the variance of the K process over this thickness.

The transient development of the spreading process does not appear to depend on the fractal dimension. Our analysis indicates that the travel distance necessary to reach the asymptotic conditions is also scale-dependent and is directly proportional to the squared characteristic thickness of the plume (L_0^2), and inversely proportional to the pore level transverse dispersivity (α_T).

In addition to analyzing the spreading of mean concentration field, we also investigated the impact of the fractal dimension of permeability distribution on the concentration fluctuations. The results indicate that the concentration variance decreases when the fractal dimension increases. This behavior is explained by revoking that the higher fractal dimension enhances the mixing process, which in turn smooths the concentration field.

It is recognized that these results were obtained for a rather idealized transport scenario of perfectly stratified media. In our opinion the results presented in this paper show the qualitative behavior of the spreading process for fractal distributions of hydraulic conductivity. We are currently working on the fully three-dimensional case with isotropic and anisotropic fractal distributions of log K. The results of this effort will be reported in the near future.

It is felt, however, that even the qualitative results presented herein lead to narrowing the gap in our understanding of subsurface transport processes, particularly regarding the actual mixing of soluble plumes with surrounding groundwater. It is a very important phenomenon which has not been given enough attention in the past. It is generally agreed that the mixing of soluble contaminant plumes with the oxygen-containing groundwater is, along with the actual aerobic biodegradation process, the major mechanism contributing to and limiting the biodegradation of soluble hydrocarbon plumes (see, for example, *Frind et al.* [1989]). Our results clearly indicate that there is a strong connection between the fractal dimension of K and the mixing process.

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