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THE STUDY OF α IN GM(1,1) MODEL

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ABSTRACT

The purpose of this paper is to analyze a predicted error in using the GM(1, 1) model based on the parameter α. The transfer function for the predicted error with the parameter α in the GM(1, 1) model is presented. The algorithm of solving equations in calculus is used to analyze whether the α is adaptive or not. The criterion of α is applied to describe the adaptive criterion of α. Finally, an example of the cage-net amounts of fish in the Peng-hu area is used to demonstrate the small prediction error due to the optimal α value. The result shows that the criterion for α is applicable for minimizing the predicted error easily.

I. INTRODUCTION

In a traditional prediction, there are many time-series analysis methods, such as the Box Jenkin's method, Holt's exponential, Winter exponential, the Regression method, and the Causal regression method (Yang *et al.*, 1995), which are used in this research field. These methods use curve fitting to perform extrapolations. It can therefore be concluded that the traditional methods can be divided into three groups: short-term, mid-term and long-term (Wen *et al.*, 1999).

According to the citations above, in a real system, information is always insufficient. Therefore, the traditional methods have not only low accuracy in making predictions, but they also need numerous amounts of data to do the predictions. So, if too few data is inputted, then the prediction may contain a large margin of error. This motivates us to present a new method, a Grey system, to decrease prediction errors for extrapolation.

Deng (1989) proposed the Grey system theory. According to the characteristics of the Grey system theory, we know that it has been extensively applied to various fields, such as data processing, modeling, control, prediction, system analysis, and decision making (Deng, 1989). In addition, the Grey system theory has two important characteristics (Huang and Yu, 1997):

1. It can estimate the unknown system using only four amounts of data.
2. It can characterize the behavior of the unknown system by using a first-order differential equation.

Therefore, Grey system theory is applied in this study to minimize the prediction error during the extrapolation process.

II. THEORETICAL ANALYSIS

A system is known as a Grey system when a part

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of the system's information is unknown. A Grey system can be described with the Grey dynamic prediction models (GDPM). Also, when the system is described with H variables and an Nth-order differential equation, it is known as a GM(N, H) model. Therefore, the GM(1, 1) consists of only one variable and the first-order differential equation (Deng, 1989). The GM(1, 1) model is therefore the simplest and easiest model in the Grey models. Since the advantage of the GM(1, 1) model for predicting the future characteristics of the system requires few known data (3 to 7 points), the GM(1, 1) model was used for predicting ground water (Wen et. al., 2000). Fundamental algorithms of the model are presented as follows.

1. GM(1, 1) Model

An original sequence of data with n measurements is expressed as

$$x^{(0)}=\{x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n)\}, \tag{1}$$

where the superscript (0) of $x^{(0)}(i)$ represents the original sequence. After the first-order accumulated generating operation (AGO) is applied to Eq. (1), it leads to

$$x^{(1)}=\{x^{(1)}(1), x^{(1)}(2), x^{(1)}(3), \ldots, x^{(1)}(n)\}, \tag{2}$$

where $x^{(1)}(k)=\sum_{i=1}^{k} x^{(0)}(i)$ for $k=1, 2, \ldots, n$. The MEAN generating operation is defined as , for

$$Z^{(1)}(k)=\alpha x^{(1)}(k)+(1-\alpha)x^{(1)}(k-1), \quad k=2, 3, \ldots, n, \tag{3}$$

where α represents the parameter for the MEAN generating operation and is in the range of [0,1] (Deng, 1989). By taking the MEAN generating operation (as defined in Eq. (3)) for Eq. (2), it leads to

$$Z^{(1)}=\{Z^{(1)}(2), Z^{(1)}(3), \ldots, Z^{(1)}(n)\}, \tag{4}$$

The Grey model GM(1,1) of $x^{(1)}(t)$ is described by the following differential equation,

$$\frac{dx^{(1)}(t)}{dt}+ax^{(1)}(t)=b\ , \tag{5}$$

where the unknown parameters, a and b, can be determined by adopting the least-square method in solving the Grey-difference equations. The Grey-difference equations are established as (Deng, 1989),

$$X^{(0)}(k)+aZ^{(1)}(k)=b, \text{ for } k=2, 3, \ldots, n, \tag{6}$$

which can be expressed in the following matrix form:

$$y_n=B\hat{a}\ , \tag{7}$$

where $y_n=\begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}$, $B=\begin{bmatrix} -Z^{(1)}(2) & 1 \\ -Z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -Z^{(1)}(n) & 1 \end{bmatrix}$, $\hat{a}=\begin{bmatrix} a \\ b \end{bmatrix}$.

With the least-square method, the solution of Equation (7) is

$$\hat{a}=\begin{bmatrix} a \\ b \end{bmatrix}=(B^TB)^{-1}B^Ty_n\ . \tag{8}$$

Let $\hat{x}^{(1)}(k)$ denote the prediction sequence of $x^{(1)}(k)$. The sequence can be solved by using Eq. (5). It can be expressed as

$$\hat{x}^{(1)}(k+1)=[x^{(0)}(1)-\frac{b}{a}]e^{-ak}+\frac{b}{a}, \quad k=1, 2, 3, \ldots \tag{9}$$

By applying the first-order inverse accumulated generating operation (IAGO) to Eq. (9), the prediction data of original sequences, $\hat{x}^{(0)}(k)$, can be obtained as

$$\hat{x}^{(0)}(k)=\hat{x}^{(1)}(k)-\hat{x}^{(1)}(k-1), \quad k=2, 3, \ldots \tag{10}$$

Therefore, the prediction data can be expressed by combining Eqs. (9) and (10) as

$$\hat{x}^{(0)}(k)=[\hat{x}^{(0)}(1)-\frac{b}{a}][1-e^{a}]e^{-a(k-1)}, \tag{11}$$

where $k=2, 3, \ldots, n-1$.

Now, we will establish an error formula for the predicted value of $x^{(0)}(k)$:

$$e(k)=\left|\frac{x^{(0)}(k)-\hat{x}^{(0)}(k)}{x^{(0)}(k)}\right|\times 100\%, \tag{12}$$

where $e(k)$ denotes the error between the predicted value of $x^{(0)}(k)$ and the true value of $x^{(0)}(k)$. The $\hat{x}^{(0)}(k)$ represents the predicted value of $x^{(0)}(k)$. Eq. (12) can be used to estimate the error between the predicted value and the true value of $x^{(0)}(k)$.

2. The Analysis of α

According to previous research of the GM(1,1) model (Yang *et al.*, 1995), the α always equals 0.5. However, with α= 0.5, the predicted error may be too large to be accepted. Although some papers have presented the excessive error problem (Lou and Zhang, 1991; Yang, 1994; Yeh and Lu, 1996), the α still was assumed as 0.5 in their studies. Therefore, based on the characters of α, we began to build a closed form of solving α with the predicted error of $x^{(0)}(k)$.

First, by substituting Eq. (11) into Eq. (12), it yields

$$e(k)=\left|\frac{x^{(0)}(k)-(1-e^{a})[x^{(0)}(1)-\frac{b}{a}]e^{-a(k-1)}}{x^{(0)}(k)}\right|\times 100\%$$

$$k\geq 2. \tag{13}$$

Since in Eq. (13) the denominator is a known value, we only need to analyze the value of the numerator. Therefore, by taking into consideration the numerator of Eq. (13) by performing some simple operations, a new error formula can be reached:

$$\hat{e}(k)=\left|x^{(0)}(k)-x^{(0)}(1)e^{-a(k-1)}+\frac{b}{a}e^{-a(k-1)}+x^{(0)}(1)e^{-a(k-2)}\right.$$

$$\left.-\frac{b}{a}e^{-a(k-2)}\right|. \tag{14}$$

Equation (14) is corresponding to α. Using calculus, the minimum error for $\alpha\in[0,1]$ can be found by taking a differential for Eq. (14) with respects to α

$$\frac{d\hat{e}(k)}{d\alpha}=0+(k-1)x^{(0)}(1)e^{-a(k-1)}\frac{da}{d\alpha}-\frac{1}{a^2}be^{-a(k-1)}\frac{da}{d\alpha}$$

$$+\frac{1}{a}e^{-a(k-1)}\frac{db}{d\alpha}-(k-1)\frac{b}{a}e^{-a(k-1)}\frac{da}{d\alpha}$$

$$-(k-2)x^{(0)}(1)e^{-a(k-2)}\frac{da}{d\alpha}+\frac{1}{a^2}be^{-a(k-2)}\frac{da}{d\alpha}$$

$$-\frac{1}{a}e^{-a(k-2)}\frac{db}{d\alpha}+(k-2)\frac{b}{a}e^{-a(k-2)}\frac{da}{d\alpha}=0 \tag{15}$$

Since Eq. (15) is not easily solved symbolically, we can use the interval-halving method to solve Eq. (15) for $\alpha\in[0,1]$. In other words, we can say that α is adaptive in the intervals of 0 and 1.

3. The Criterion of α

After the adaptive α is proved, the major effects on the GM(1,1) model can be found by examining Eq. (3) carefully. We utilize the criterion of α to find the optimal value of α to minimize the error in the GM(1,1) model.

The criterion of α is built based on the definition of the class ratio in the Grey theory (Deng, 1989):

$$\sigma^{(1)}(k)=\frac{x^{(1)}(k-1)}{x^{(1)}(k)} \quad k=2, 3, 4, \ldots, n\in N \tag{16}$$

Using the concept of the ratio of the whole trend to an individual trend, a new class ratio for the criterion of α can be written as

$$\hat{\sigma}^{(1)}(k)=\frac{x^{(1)}(k)-x^{(1)}(k-1)}{\dfrac{x^{(1)}(n)-x^{(1)}(1)}{n-1}} \quad k=2, 3, 4, \ldots, n\in N \tag{17}$$

where $\hat{\sigma}^{(1)}(k)$ represents the new class ratio between the individual trend and the whole trend.

In order for the GM(1,1) model to be used to predict the sequence $x^{(0)}(k)$, we take n data from the sequence $x^{(0)}(k)$ to analyze them. Eq. (17) can therefore be rewritten as

$$\hat{\sigma}=\frac{[n-1]\times[x^{(1)}(k)-x^{(1)}(k-1)]}{x^{(1)}(n)-x^{(1)}(1)}=P_k. \tag{18}$$

It is worth noting that P_k is always positive due to the sequence of $x^{(1)}(k)$ being a monotonic increase.

Rearranging Eq. (18) reaches:

$$(n-1)\times[x^{(1)}(k)-x^{(1)}(k-1)]=P_k\times[x^{(1)}(n)-x^{(1)}(1)]. \tag{19}$$

As k=2, 3, 4, ..., n, Eq. (19) becomes

$$(n-1)\times[x^{(1)}(2)-x^{(1)}(1)]=P_2\times[x^{(1)}(n)-x^{(1)}(1)]$$

$$(n-1)\times[x^{(1)}(3)-x^{(1)}(2)]=P_3\times[x^{(1)}(n)-x^{(1)}(1)]$$

$$(n-1)\times[x^{(1)}(4)-x^{(1)}(3)]=P_4\times[x^{(1)}(n)-x^{(1)}(1)],$$

$$\vdots$$

$$(n-1)\times[x^{(1)}(n)-x^{(1)}(n-1)]=P_n\times[x^{(1)}(n)-x^{(1)}(1)] \tag{20}$$

With the definition of AGO, the relationship between the original sequence and first generating sequence can be written as ,

$$[x^{(1)}(2)-x^{(1)}(1)]=x^{(0)}(2),$$

$$[x^{(1)}(3)-x^{(1)}(2)]=x^{(0)}(3),$$

$$[x^{(1)}(4)-x^{(1)}(3)]=x^{(0)}(4),$$

$$\vdots$$

$$[x^{(1)}(n)-x^{(1)}(n-1)]=x^{(0)}(n), \tag{21}$$

Taking a summation of all the equations in Eq. (21) yields

$$[x^{(1)}(n)-x^{(1)}(1)]=(\sum_{k=2}^{n}x^{(0)}(k)). \tag{22}$$

Substituting Eqs. (21) and (22) into Eq. (20) yields

Table 1. The fish amounts from 1993 to 1998 in Peng-hu area

Year	1993	1994	1995	1996	1997	1998
Amounts (100tons)	5.08	6.25	8.79	12.52	15.61	18.51

$$(n-1)\times[x^{(0)}(2)]=p_2\times[x^{(0)}(2)+x^{(0)}(3)+x^{(0)}(4)+\ldots+x^{(0)}(n)],$$

$$(n-1)\times[x^{(0)}(3)]=p_3\times[x^{(0)}(2)+x^{(0)}(3)+x^{(0)}(4)+\ldots+x^{(0)}(n)],$$

$$(n-1)\times[x^{(0)}(4)]=p_4\times[x^{(0)}(2)+x^{(0)}(3)+x^{(0)}(4)+\ldots+x^{(0)}(n)],$$

$$\vdots$$

$$(n-1)\times[x^{(0)}(n)]=p_n\times[x^{(0)}(2)+x^{(0)}(3)+x^{(0)}(4)+\ldots+x^{(0)}(n)], \qquad (23)$$

To solve the value of each p_i (i=1, 2, 3, ..., n) and to use p_i=1 as a threshold value for the α criterion, the general rule of p_i can be found by using the data characteristics where Eq. (17) presents the ratio of the increment of two neighborhood points to the average increment between the initial (i=1) and final points (i=n). The general rules of the α criterion therefore can be written as:

1. When p>1, it means the second term on the right-hand side in Eq. (3) has more significance than the first term on the right-hand in Eq. (3), then we can say that $\alpha\rightarrow 0.00$
2. When p<1, the second term on the right-hand side in Eq. (3) has less important than the first term on the right-hand in Eq. (3), then we can say $\alpha\rightarrow 1.00$

III. AN EXAMPLE OF PREDICTION

According to the Grey system characteristics, we can use the initial four data methods to predict the fifth datum in the system (Yu, 1989; Huang *et al.*, 1997). Furthermore, we can use the prediction principle to prove that α is adaptive to the Grey system.

A data set of cage-net amounts of fish in the Peng-hu area from 1993 to 1998 is provided by Mr. Jing-Yi Pan (Pan, 2000) and shown in Table 1.

Since the data is given as {5.08, 6.25, 8.97, 12.52, 15.61, 18.51} which shows no fluctuation in the data set, we use the difference between the consequence data as our original sequence for the GM(1,1) system,

$$x^{(0)}=\{1.17, 2.72, 3.55, 3.09, 2.9\}.$$

In order to use the first four data to predict the fifth one, we have the AGO of $x^{(0)}(k)$ as

$$x^{(1)}=\{1.17, 3.89, 7.44, 10.53\}.$$

By substituting $x^{(0)}(k)$ and $x^{(1)}(k)$ into Eq. (1), we can obtain

$$Y=\begin{bmatrix}2.72\\3.55\\3.09\end{bmatrix},\quad B=\begin{bmatrix}-2.72\alpha-1.17 & 1\\-3.55\alpha-3.892 & 1\\-3.09\alpha-7.44 & 1\end{bmatrix}$$

By taking the transpose of B as

$$B^T=\begin{bmatrix}-2.72\alpha-1.17 & -3.55\alpha-3.89 & -3.09\alpha-7.44\\1 & 1 & 1\end{bmatrix}.$$

According to $\hat{\alpha}=(B^TB)^{-1}B^TY$, the values of a and b are

$$a=\frac{-1.0374\alpha-2.9445}{1.0374\alpha^2+5.889\alpha+59.3138},$$

$$b=\frac{4.8322\alpha+158.2733}{1.0374\alpha^2+5.889\alpha+59.3138}.$$

Let k=2, 3, and 4 for Eq. (12), we find ,

$$\frac{d\hat{e}(2)}{d\alpha}=1.17e^{-a}\frac{da}{d\alpha}-\frac{1}{a^2}be^{-a}\frac{da}{d\alpha}+\frac{1}{a}e^{-a}\frac{db}{d\alpha}-\frac{b}{a}e^{-a}\frac{da}{d\alpha}+\frac{b}{a^2}\frac{da}{d\alpha}-\frac{1}{a}\frac{db}{d\alpha},$$

$$\frac{d\hat{e}(3)}{d\alpha}=2.34e^{-2a}\frac{da}{d\alpha}-\frac{b}{a^2}e^{-2a}\frac{da}{d\alpha}+\frac{1}{a}e^{-2a}\frac{db}{d\alpha}-2\frac{b}{a}e^{-2a}\frac{da}{d\alpha}-1.17e^{-a}\frac{da}{d\alpha}+\frac{b}{a^2}e^{-a}\frac{da}{d\alpha}-\frac{1}{a}e^{-a}\frac{db}{d\alpha}+\frac{b}{a}e^{-a}\frac{da}{d\alpha},$$

$$\frac{d\hat{e}(4)}{d\alpha}=3.51e^{-3a}\frac{da}{d\alpha}-\frac{b}{a^2}e^{-3a}\frac{da}{d\alpha}+\frac{1}{a}e^{-3a}\frac{db}{d\alpha}-3\frac{b}{a}e^{-3a}\frac{da}{d\alpha}-2.34e^{-2a}\frac{da}{d\alpha}+\frac{b}{a^2}e^{-2a}\frac{da}{d\alpha}-\frac{1}{a}e^{-2a}\frac{db}{d\alpha}+2\frac{b}{a}e^{-2a}\frac{da}{d\alpha},$$

where

$$\frac{da}{d\alpha}=\frac{(1.037\alpha+2.9445)\times(2.0748\alpha+5.889)}{(1.0374\alpha^2+5.889\alpha+59.3138)^2}-\frac{1.037}{1.0374\alpha^2+5.889\alpha+59.3138},$$

$$\frac{db}{d\alpha}=\frac{(2.0748\alpha+5.889)\times(4.8322\alpha+158.2733)}{(1.0374\alpha^2+5.889\alpha+59.3138)^2}+\frac{4.8322}{1.0374\alpha^2+5.889\alpha+59.3138},$$

From the above equations, by using the interval-halving method,

$$\left\{\frac{d\hat{e}(2)}{d\alpha}\right\}_{\alpha=0} \times \left\{\frac{d\hat{e}(2)}{d\alpha}\right\}_{\alpha=1} > 0,$$

$$\left\{\frac{d\hat{e}(3)}{d\alpha}\right\}_{\alpha=0} \times \left\{\frac{d\hat{e}(3)}{d\alpha}\right\}_{\alpha=1} > 0,$$

$$\left\{\frac{d\hat{e}(4)}{d\alpha}\right\}_{\alpha=0} \times \left\{\frac{d\hat{e}(4)}{d\alpha}\right\}_{\alpha=1} > 0,$$

Since we can not directly solve α in the range [0, 1] to minimize the predicted error between the predicted and true values of $x^{(0)}(k)$, we can say that α is adaptive. Fig. 1 also shows the evidence that the values of $\frac{d\hat{e}(2)}{d\alpha}$, $\frac{d\hat{e}(3)}{d\alpha}$, and $\frac{d\hat{e}(4)}{d\alpha}$ are less than zero for α in the range [0, 1]. Therefore, it shows that there are no absolute minimum errors for $\frac{d\hat{e}(2)}{d\alpha}$, $\frac{d\hat{e}(3)}{d\alpha}$, and $\frac{d\hat{e}(4)}{d\alpha}$ and that all are less than zero. We apply the α criterion to find the optimal α to make the error approach minimum. Therefore, Eq. (23) can be used for solving p_i (i=2, 3, 4):

$$p_2 = \frac{3\times(3.89-1.17)}{(10.53-1.17)} = 0.8718 < 1,$$

$$p_3 = \frac{3\times(7.44-3.89)}{(10.53-1.17)} = 1.1378 < 1,$$

$$p_4 = \frac{3\times(10.53-7.44)}{(10.53-1.17)} = 0.9904 < 1.$$

Hence, using the value of α=0.01 for Eq. (16) yields

a=-0.049642748, b=2.668406003.

By using the values a and b, and based on Eq. (7), we can say that the fifth datum in our example would be

$$\hat{x}^{(0)}(5) = (1 - e^{-0.049642748}) \times [1.17 + \frac{2.668406003}{0.049642748}] \times e^{4\times 0.049642748} = 3.24419303$$

If we use the value of α=0.99 and substitute it into Eq. (16), then

a=-0.060028507, b=2.464563743

By using the values a and b, based on Eq. (7),

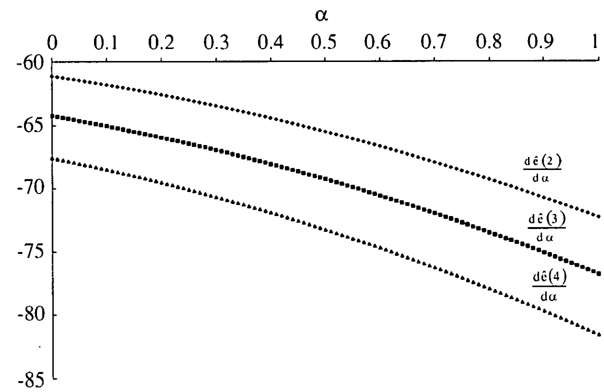

Fig. 1 The values of $\frac{d\hat{e}(2)}{d\alpha}$, $\frac{d\hat{e}(3)}{d\alpha}$, and $\frac{d\hat{e}(4)}{d\alpha}$ related to the α

we can say that the fifth datum in our example would be

$$\hat{x}^{(0)}(5) = (1 - e^{-0.060028507}) \times [1.17 + \frac{2.464563743}{0.060028507}] \times e^{4\times 0.060028507} = 3.127905119$$

If we use the value of α=0.5 and substitute it into Eq. (16), then

a=-0.055395556, b=2.570304546,

and the fifth datum in our example would be

$$\hat{x}^{(0)}(5) = (1 - e^{-0.055395556}) \times [1.17 + \frac{2.570004546}{0.055395556}] \times e^{4\times 0.055395556} = 3.199325486$$

From the original data, the predicted error for the difference of α are:

1. For α=0.01, the error is

$$\hat{e}_{\alpha=0.01} = \frac{|2.9 - 3.24419303|}{2.9} \times 100\% = 11.87\%,$$

2. For α=0.5, the error is

$$\hat{e}_{\alpha=0.5} = \frac{|2.9 - 3.199325486|}{2.9} \times 100\% = 10.32\%,$$

3. For α=0.99, the error is

$$\hat{e}_{\alpha=0.99} = \frac{|2.9 - 3.127905119|}{2.9} \times 100\% = 7.86\%,$$

In the last three estimated formulae, it shows that for α=0.99 there would be relatively small prediction errors. This again can be explained by using the figure of $\frac{d\hat{e}(5)}{d\alpha}$. The result is illustrated in Fig. 2.

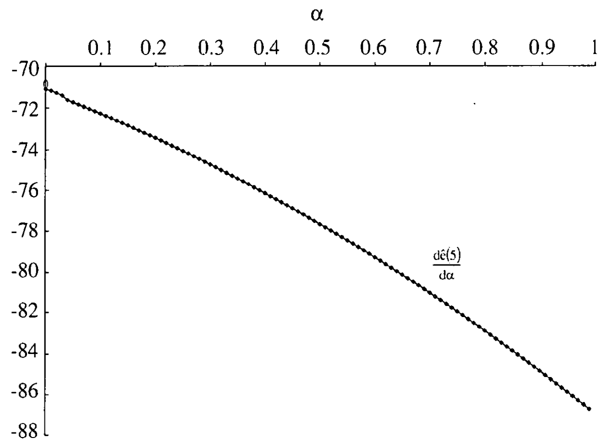

Fig. 2 The value of $\frac{d\hat{e}(5)}{d\alpha}$ related to the α

In Fig. 2, we can see that the small value of $\frac{d\hat{e}(5)}{d\alpha}$ is at $\alpha \to 1$. It shows that there exists a relatively small prediction error of $\hat{e}(5)$ for α in the range of [0,1] as $\alpha \to 1$.

IV. CONCLUSIONS

In this paper, we presented the approaches of GM(1,1), such as the concept of the transfer function for the $\frac{d\hat{e}(k)}{d\alpha}$, α in the GM(1,1) mode, and the adaptive of α in [0,1].

Meanwhile, a criterion for adaptive α is presented. Also from predicted-error analysis, the overall predicted error can be reduced sooner under this criterion. Finally, using the criterion of α to decide the optimal value of α has both theoretical and practical possibilities. Specifically, for the example of the fish in the Peng-hu area, it shows that the result, the predicted error being 7.86% at α=0.99, is quite acceptable. However, since we only presented one example for demonstrating our findings, we feel that more research on this topic should be conducted in the future.

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NOMENCLATURE

$x^{(0)}(i)$	represents the original sequence, i=1, 2, 3, ..., n.
$x^{(1)}(i)$	the first-order accumulated generating operation (AGO).
α	the parameter for the MEAN generating operation.
$Z^{(1)}(i)$	the MEAN generating operation.
a and b	the unknown parameters of the Grey-difference equations.
$\hat{x}^{(1)}(i)$	the prediction sequence of $x^{(1)}(i)$.
$\hat{x}^{(0)}(i)$	the prediction data of original sequence, $x^{(0)}(i)$.
$e(k)$	the error between the predicted value of $x^{(0)}(k)$ and the true value of $x^{(0)}(k)$.
$\sigma^{(1)}(i)$	the class ratio.
$\hat{\sigma}^{(1)}(i)$	the new class ratio between the individual trend and the whole trend.

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Discussions of this paper may appear in the discussion section of a future issue. All discussions should be submitted to the Editor-in-Chief.

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GM(1,1)模型中 α 分析

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摘　要

本文利用GM(1,1)模型中的α值對預測誤差大小進行分析，藉由詳細介紹GM(1,1)模型的方式找出 α 乃是影響誤差大小的主要因素及導出預測值之誤差和α值的關係式。並根據微積分證明α為適應的，隨即提出α準則做為調整α的依據，使預測誤差降至最低。最後，則以一實際例證做為本文提出之理論的驗證，由實例中可以了解本文所提出的α準則應是適合於最小預測誤差要求時做為最佳 α 值之決定使用。

關鍵字：GM(1,1)模型，預測誤差，適應。