A STUDY OF MEAN AREAL PRECIPITATION AND SPATIAL STRUCTURE OF RAINFALL DISTRIBUTION IN THE TSEN-WEN RIVER BASIN

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Key Words: thiessen method, kriging method, stochastic approach, semivariogram.

ABSTRACT

In this paper, the kriging method with stochastics was applied in order to study the spatial structure of precipitation distributions. Mean areal precipitation and the spatial structure of six storms over the Tsen-Wen River basin were examined by semivariograms of the storms.

Findings revealed topographic influences on the rainfall distributions in the study area. These impacts were reflected in the difference between the mean areal precipitation of the kriging and the Thiessen methods and the correlation structure (or range) of the rainfall distributions. The mean areal precipitation showed a difference of 2.5% in favor of the kriging method. The range of the rainfall distribution averaged about 46.17-km, a very close correlation to the downstream length, 47-km, of the Tsen-Wen River. In addition, the smaller the standard deviations of the precipitation distributions, the more appropriate it is to use the assumption of the second-order stationarity for investigating the storm structures.

I. INTRODUCTION

The evaluation of mean seasonal or annual rainfall over basins is common for water-balance research. Daily (or hourly) areal averages and evaluations of storms are necessary in flood forecasting, specifically, for real-time reservoir operations or model calibration of rainfall-runoff models. Under homogeneous distribution of rainfall in river basins, many methods for the estimation of mean areal precipitation have been developed over the past few decades. The methods include the Thiessen polygons, arithmetic, and ilsohyeta methods (Chow, 1964; Linsley *et al*., 1975). However, applying these methods to a large river basin relies on a tremendous amount of paper work and manpower. Also, not investigated in these methods are the impacts from the topographic variations of a basin on the estimates of the mean areal precipitation.

Due to spatial patterns of topographic variation in the study area, spatial rainfall distributions are considered as random fields so that the distributions can be studied by means of stochastic approaches over the field (Mejia and Rodriguez-Iturbe, 1974; Bras and Rodriguez-Iturbe, 1985; Chua and Bras, 1980). The method for using stochastic approaches for random fields is by analyzing the spatial structure of the fields (de Marsily, 1986; Bakr, 1976; Gelhar, 1986; Kemblowski and Wen, 1993). However, the stochastic approaches are used indirectly to obtain estimates of the mean areal precipitation. The estimate of the mean areal precipitation is extracted from within the results of the stochastic equations.

Kriging techniques provide linear estimates of minimal variance. Chua and Bras (1980) and de Marsily (1986) used them with stochastic approaches. The kriging techniques were based on the concept of linear interpolation and the average rainfall over the concerned basins. This combination was used to obtain the mean areal precipitation. During the estimation of the mean areal precipitation, stochastic approaches with semivariograms were involved in the kriging techniques as estimators. However, unlike Chua and Bras and de Marsily, most researchers (Delhomme, 1979; Delfiner and Delhomme, 1973; David, 1976) have not attempted to investigate the spatial structure of random fields. For the investigation of spatial structure, covariances of random fields were used (Meija and Rodriguez-Iturbe, 1974; Bakr, 1976; Gelhar, 1986; Kemblowski and Wen, 1993).

There are two ways to derive covariances of random distributions. The first way is to obtain spectrums of the rainfall distributions, and then to find the covariances with the Wiener-Khintchine relation by integral schemes. The second way is to obtain the covariance directly from the data of random fields. The latter way is easier in practice than the former. Since two filters are involved in the first method, spectrums of random processes and the Wiener-Khintchine relation, covariances derived from the first method have less noise than the second. For practical reasons, the first one has many more mathematical procedures to complete to derive the covariance than the second. Therefore, in this paper, the latter procedure, due to its simplicity, is used for the estimation of the semivariograms in the kriging method.

In addition, the second-order stationarity hypothesis can be applied where the mean is constant and the covariance is a function of the distance between neighboring points only. By applying the second-order stationarity to rainfall distributions, the semivariograms of the rainfall distributions are therefore able to be equal to the variance of the distributions minus the covariance of the distributions when an extra equation is included. Consequently, it is possible to investigate spatial structures of rainfall distributions with semivariograms.

The aim of this paper is to employ the kriging technique with stochastic approaches to obtain estimates of the mean areal precipitation over the Tsen-Wen River basin on a pacific island, Taiwan. Concepts of the kriging method in Chua and Bras (1980) and de Marsily (1986) are combined with the estimates of the mean areal precipitation in this paper.

The important point of this article is that the estimates of the mean areal precipitation and the investigation of spatial structures of rainfall distributions are accomplished by estimates derived from the semivariograms of the rainfall distributions over the Tsen-Wen River basin. One difference (from the work of Chau & Bras and de Marsily) is that they used the second-order stationarity hypothesis by including an additional equation to arrive at semivariograms. In this paper, reference will be made to another hypothesis known as the intrinsic hypothesis. To arrive at semivariograms, this hypothesis will replace the second-order stationarity, since the intrinsic hypothesis includes the semivariogram equation directly without requiring additional calculations.

II. MATHEMATICAL DESCRIPTION OF THE KRIGING METHOD

In general, kriging is a method for optimizing the estimate of a magnitude, which is distributed in space and is measured at a network of points. Let $\underline{u}_1$, $\underline{u}_2$, ..., and $\underline{u}_n$ be the locations of the n observation points and $\underline{u}_i$ denote a two-dimensional representation. Also, let $Z(\underline{u}_i)$ be the value (such as: precipitation) measured at point i. The problem with the estimation of a point lies in determining the value of the quantity $Z(\underline{u}_0)$ for any point $\underline{u}_0$ that has not been measured. By continually modifying the position of the point $\underline{u}_0$, it is thus possible to estimate the whole field of the parameter. De Marsily (1986) gives an excellent review of the kriging method.

This study deals with random distributions of rainfall to obtain the mean areal precipitation. Previously the second-order stationarity used with the kriging method was mentioned. This method involves the mean of the rainfall distribution and the covariance of the distance between points only. However, a more reliable method, known as the intrinsic hypothesis can also be used when the random distributions of rainfall are stationary (Matheron, 1970; de Marsily, 1986). As mentioned previously, this hypothesis includes the semivariogram equation directly without making any additional calculations, such as are required by the second-order stationarity to arrive at semivariograms.

There are two conditions for the intrinsic hypothesis which $Z(\underline{u}_1)-Z(\underline{u}_2)$ satisfies:

$$E[Z(\underline{u}_1)-Z(\underline{u}_2)]=m(\underline{h}), \tag{1}$$

$$Var[Z(\underline{u}_1)-Z(\underline{u}_2)]=2\gamma(\underline{h}), \tag{2}$$

where

$m(\underline{h})$ represents the mean of $Z(\underline{u}_1)-Z(\underline{u}_2)$ and is a function of $\underline{h}$ only,

$\gamma(\underline{h})$ denotes the semivariogram of rainfall distributions $Z(\underline{u}_i)$ and is a function of $\underline{h}$ only,

$Var[Z(\underline{u}_1)-Z(\underline{u}_2)]$ represents the variance of $Z(\underline{u}_1)-Z(\underline{u}_2)$,

$Z(\underline{u}_i)$ denotes the measured rainfall at observation points $\underline{u}_i$,

and

$\underline{h}$ is equal to $\underline{u}_1-\underline{u}_2$.

Another name for $m(\underline{h})$ is "drift". A simple way to estimate the drift, mathematically, is to assume that the drift has a linear relation with h and is expressed as

$$E[Z(\underline{u}_1)-Z(\underline{u}_2)]=a\underline{h},$$

where a is some constant. However, in practical applications, the drift is sometimes assumed to be constant (i.e., a=0, and called "no drift") in which case the semivariograms can be estimated directly from the observations $Z(\underline{u}_i)$ without estimating the linear drift coefficient, a, beforehand.

By the way, the operator in Eq. (1), $E[\ ]$, denotes taking expectations of random distributions $Z(\underline{u}_1)-Z(\underline{u}_2)$. In this paper, rainfall distributions in the Tsen-Wen River basin are assumed to be stationary. The drift is therefore zero. The expected value of rainfall distributions Z can be derived from Eq. (1) as

$$E[Z(\underline{u}_1)]=E[Z(\underline{u}_2)]=m=\text{constant}. \tag{3}$$

It is apparent in Eq. (3) that the mean of rainfall distributions is constant, known as no drift of rainfall distributions. Therefore, the semivariograms can be estimated directly from the observations $Z(\underline{u}_i)$ without the need to estimate the linear coefficient, a.

For the investigation of spatial structures of rainfall distributions with the second-order stationarity, the covariance of rainfall distributions can be written as (de Marsily, 1986)

$$E[Z(\underline{u}_1)-Z(\underline{u}_2)]=\sigma^2-\gamma(\underline{u}_1-\underline{u}_2), \tag{4}$$

where $E[Z(\underline{u}_1)\ Z(\underline{u}_2)]$ denotes the covariance function of the rainfall distribution between $\underline{u}_1$ and $\underline{u}_2$, and σ^2 denotes the variance of rainfall distribution. In this case, the variance σ^2 has to be finite (Eq. (2)) so that the rainfall distributions satisfy the second-order stationarity assumption.

One of the interesting quantities in this paper is the mean areal precipitation over some area of interest S_0. The true unknown mean areal precipitation, denoted Z_{S_0}, can be expressed as

$$Z_{s_o}=\frac{1}{S_o}\int_{S_o} Z(\underline{u})d\underline{u}. \tag{5}$$

The objective of this work is to find the BLUE (the best linear unbiased estimate), $Z^*_{s_o}$, of the true unknown value, Z_{S_0}. The BLUE, also known as kriging, is defined as (Chua and Bras, 1980):

(i) Linear: The estimator, $Z^*_{s_o}$, is formulated with a linear combination of the measured values of $Z(\underline{u}_i)$:

$$Z^*_{s_o}=\sum_i^n \lambda_i Z(\underline{u}_i), \tag{6}$$

where λ_i represents the optimal weight at location $\underline{u}_i$.

(ii) Unbiased: Theoretically, it is required that the expected value of the estimator, $Z^*_{s_o}$, has to equal the expected value of the true unknown mean areal precipitation, Z_{S_0}, i.e.,

$$E[Z^*_{s_o}]=E[Z_{s_o}]. \tag{7}$$

(iii) Best criterion: If the estimated variance of the estimator, $Z^*_{s_o}$, were minimal, it would be considered best. The estimated variance, called the mean square error (MSE), is defined as

$$\sigma^2_{Z_{s_o}}=Var[(Z_{s_o}-Z^*_{s_o})]=E[(Z_{s_o}-Z^*_{s_o})^2]. \tag{8}$$

Under the assumptions that the semivariogram is known and the mean is stationary (no drift), the BLUE condition is satisfied. Therefore, substituting Eqs. (5) and (6) into Eq. (7), yields

$$m[\sum_{i=1}^n \lambda_i-1]=0. \tag{9}$$

In Eq. (9), m denotes the mean areal precipitation and does not have to be zero. Consequently,

$$\sum_{i=1}^n \lambda_i=1. \tag{10}$$

As indicated in Eq. (10), the sum of the optimal weight equals one. Then, the equations for BLUE with Eq. (10) can be used for obtaining the optimal weight (λ_i) with the multiplier of the Lagrangian method. Therefore, a system of kriging in matrix form is reached as follows:

$$\begin{pmatrix} 0 & 0 & \gamma_{13} & \cdots & \gamma_{1n} & 1 \\ \gamma_{21} & 0 & \gamma_{23} & \ddots & \ddots & 1 \\ \gamma_{31} & \gamma_{32} & 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \gamma_{n1} & \gamma_{n2} & \gamma_{n3} & \ddots & 0 & 1 \\ 1 & 1 & 1 & \cdots & 1 & 0 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \vdots \\ \lambda_n \\ \mu \end{pmatrix} = \begin{pmatrix} \overline{\gamma_{io}} \\ \overline{\gamma_{2o}} \\ \overline{\gamma_{3o}} \\ \vdots \\ \overline{\gamma_{no}} \\ 1 \end{pmatrix} \tag{11}$$

and the estimated variance is

$$\sigma^2_{Z_{s_o}}=\sum_{i=1}^n \lambda_i\overline{\gamma_{io}}+\mu-\overline{\overline{\gamma_{12}}}, \tag{12}$$

where the semivariograms used in Eqs. (11) and (12) are known, as below:

$$\overline{\gamma_{io}}=\frac{1}{S_o}\int_{S_o}\gamma(\underline{u}_i-\underline{u})d\underline{u}, \tag{13}$$

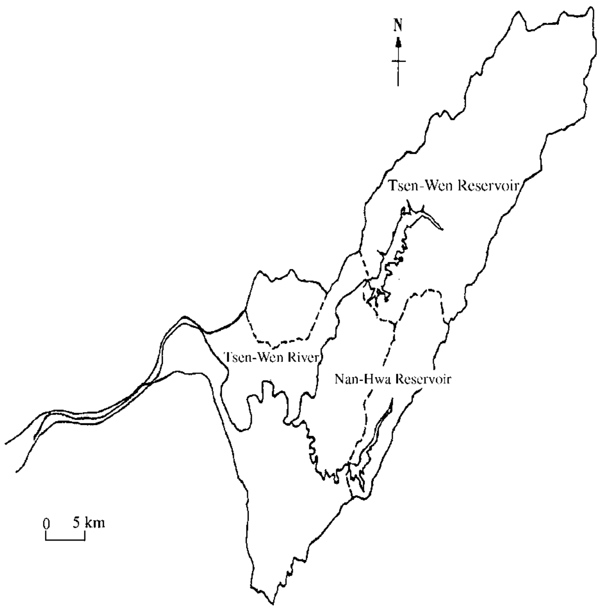

Fig. 1 Tsen-Wen River basin (scale 1:500000)

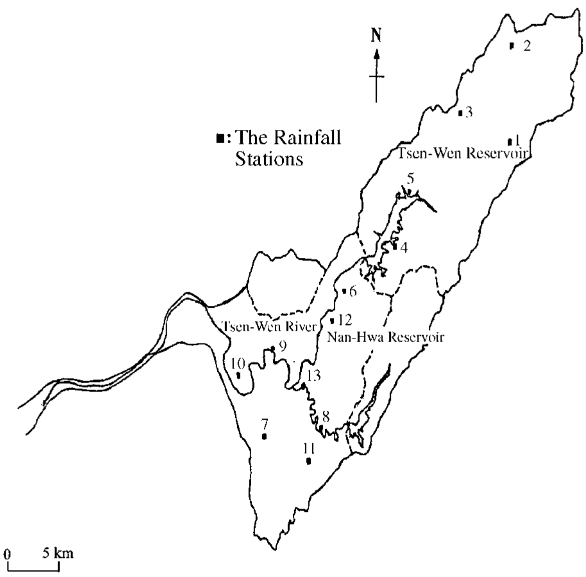

Fig. 2 Rainfall stations in Tsen-Wen River basin (scale 1:500000)

$$\overline{\overline{\gamma_{ij}}} = \frac{1}{s_o^2}\int_{S_o}\int_{S_o}\gamma(\underline{u}_i - \underline{u}_j)d\underline{u}_i d\underline{u}_j\ . \tag{14}$$

Equation (11) shows that the optimal weight can be solved when the semivariograms γ_{ij} and $\overline{\gamma_{io}}$ are known. Eq. (12) expresses the estimated variance of the mean areal precipitation. The estimated variance, in statistics, can be used for error investigation of the estimates of mean areal precipitation.

An important finding is that both Eqs. (11) and (12) are expressed in terms of semivariograms, which are derived by investigating the spatial structures of the rainfall distributions.

III. RAINFALL DATA IN STUDY AREA

The study area, the Tsen-Wen River basin, is in the southwest of Taiwan. Fig. 1 shows a location map of the Tsen-Wen River basin with its main rivers including two main tributaries. The catchment is over 1,190-km^2 in size, and the total length of the Tsen-Wen River is around 138.5-km. Specifically, it is 47-km in length at the downstream plain area. It goes from east to west and finally flows into the Taiwan Strait. The basin includes four reservoirs: Tsen-Wen Reservoir, Wu-Sen-Tao Reservoir, Nan-Hwa Reservoir, and Gin-Men Reservoir. Going from the largest reservoir to the smallest, respectively, the Tsen-Wen Reservoir in Taiwan is 481-km^2 in size, the Nan-Hwa Reservoir is 112-km^2, the Wu-Sen-Tao Reservoir is 60-km^2, and the Gin-Men Reservoir is 2.73-km^2.

Topographically, the catchment consists largely of mountainous areas in the east and flows into flatter flood plains in the west. The eastern end of the basin is steeper with the averaged slopes being about $\frac{1}{1000} \sim \frac{1.5}{1000}$. The average elevation of the ba sin is 100m above mean sea level. The basin has a very high precipitation density during the summer, which is usually a result of typhoons, a kind of tropical storm.

Many precipitation stations are in the Tsen-Wen basin. However, according to the brochure published by the National Central Weather Bureau (NCWB), there are 13 NCWB stations, including 11 automatic rain gauges and 2 manual rain gauges. These were selected for the study. The selected stations are government controlled and are more reliable than the other stations in the area. The other stations are privately owned and are operated for personal interest rather than for public service. The locations of the selected stations are indicated in Fig. 2.

Six storms have been chosen for the study. Three of the storms occurred in 1990 and the rest of them in 1992. All the rainfall data at the selected 13 stations are provided by NCWB. The point mean, also known as the arithmetic mean, and point variance of each storm are therefore estimated and shown in Table 1.

With an investigation of the point means and the point variances, storms No. 2 and 4 are found to have higher point means and storms No. 1, 2, and 4 are found to have larger point variances than the other storms. These results reflect unstable conditions. These unstable conditions are the result of topographic influences in the study area. Therefore, as expected, the assumption of the second-order stationarity is not appropriate to apply to storms No. 1,

Table 1 Point means and point variances of six storms

Storm No.	1	2	3	4	5	6
Date	June,22,1990 ~ June,24,1990	Aug,18,1990 ~ Aug,23,1990	Sep,07,1990 ~ Sep,10,1990	Aug,29,1992 ~ Sep,02,1992	Sep,03,1992 ~ Sep,06,1992	Sep,19,1992 ~ Sep,24,1992
Point Mean (cm)	27.60	53.20	29.30	41.39	19.70	13.10
Point variance	82.34	466.40	40.70	31.47	7.24	16.33

Table 2 Mean areal precipitation (MAP) estimated by the Thiessen method

Storm No.	1	2	3	4	5	6
MAP by Thiessen (cm)	28.08	55.23	28.69	42.31	19.65	13.60

Table 3 The weight factors of the rainfall stations obtained by the Thiessen method

Storm No.	1	2	3	4	5	6	7	8	9	10	11	12	13
Weight factor	0.0842	0.0893	0.1024	0.1173	0.0596	0.0938	0.0801	0.0693	0.0785	0.0995	0.0257	0.0599	0.0394

2, and 4 (Gutjahr *et al*., 1978).

Since mean areal precipitation is customarily estimated by the Thiessen method, it is therefore obtained initially by the Thiessen method and compared later to the kriging method. Therefore, the estimated mean areal precipitation, using the Thiessen method, for the selected six storms, is presented in Table 2. Table 3 indicates the weight factors of the selected rainfall stations estimated by the Thiessen method.

The weight factors from the Thiessen method for the selected rainfall stations reflect no impacts from the topography on the rainfall distributions. In other words, the assumption that rainfall distributions have to be uniformly distributed on a river basin has to be satisfied when the Thiessen method is employed. Here, that appears to be the case. However, when the Thiessen results are compared to that of kriging, a variation of the weight factors is found, indicating impacts from the topographic influences on the rainfall distributions.

IV. SPATIAL STRUCTURES AND MEAN AREAL PRECIPITAION ESTIMATES OF RAINFALL DISTRIBUTIONS

As mentioned earlier, an experimental semivariogram can be computed from the raw data when a stationary drift assumption is made. Biases introduced by estimating the mean are therefore avoided. In estimating the experimental semivariograms, pairs of stations were grouped according to their separation distance $|\underline{h}|$.

In the study case of irregularly spaced observation stations, stations that lie within discrete ranges are grouped together by pairs (de Marsily, 1986). In such a case, $n_{\underline{h}}$ denotes the number of paired stations that lie between some interval defined by $\underline{h}_1$ and $\underline{h}_2$. $\underline{h}$ represents an average value given by

$$\left|\,\underline{h}\,\right| = \frac{1}{n_{\underline{h}}}\sum_{\alpha=1}^{n_{\underline{h}}}\left|\,\underline{h}_{\,\alpha}\right| , \tag{15}$$

where $|\underline{h}_1|<|\underline{h}_\alpha|<|\underline{h}_2|$ and α=1, 2, ..., $n_{\underline{h}}$. In addition, isotropy is assumed for the use of Eq. (15). Table 4 indicates the distance intervals in kilometers; the average distance, $\underline{h}$, within each interval; and the number of paired stations ($n_{\underline{h}}$) within each interval.

After separation distances of stations have been evaluated, then we can calculate semivariograms of the rainfall distributions. From the observed rainfall distributions at the stations, calculations can be made by using the following equation:

$$\hat{\gamma}(\underline{h}) = \frac{1}{n_{\underline{h}}}\sum_{i=1}^{n_{\underline{h}}}\{Z(\underline{u}_{\,i} + \underline{h}) - Z(\underline{u}_{\,i})\}^2 . \tag{16}$$

Equation (16) denotes the semivariogram expression for discrete data observed at the stations. With the equation and the observed rainfall distributions at 13 stations for six storms, the semivariograms of the six storms are computed (Figs. 3~8). Consequently, the spatial structures of the rainfall distributions are investigated with the estimated semivariograms.

1. Spatial Structures of Rainfall Distributions

A prevailing model chosen for the experimental

Table 4 The number of paired stations in intervals of distances

Distance in Km	Average Distance L	No. of pairs of stations n_h
0-3	---	---
3-6	4.87	5
6-9	7.75	9
9-12	10.32	8
12-15	13.30	8
15-18	16.19	6
18-21	19.58	5
21-24	21.77	3
24-27	25.97	4
27-30	28.17	6
30-33	31.90	2
33-36	34.13	3
36-39	38.05	3
39-42	40.72	3
42-45	43.38	4
45-48	47.13	3
48-51	48.60	3
51-54	53.46	2
54-75	55.30	1

semivariograms is the spherical semivariogram (Chua and Bras, 1980; de Marsily, 1986; Journel and Huijbregts, 1989),

$$\gamma(h)=\begin{cases} C_o+C[1.5(\frac{h}{a})-0.5(\frac{h}{a})^3], & h\le a \\ C_o+C, & h>a \end{cases}, \quad (17)$$

where

C_o=the nugget effect,

C_o+C=the sill,

and

a=the range.

With the spherical semivariogram, a curve fitting method for establishing the sill, the range and nugget effect follows. The spherical semivariograms of six storms are presented in Figs. 3, 4, 5, 6, 7, and 8. In addition, the functions of the spherical semivariograms are listed in Table 5.

The estimated semivariograms of the six storms, estimated with Eq. (16), are found to vary following the application of the function of the spherical semi-variogram of Eq. (17). Specifically, the variations of the spherical semivariograms of storms No. 1, 2, and 4 are much higher than the spherical semivario-grams of the other storms. Also, as previously men-tioned, the point variances of storms No. 1, 2, and 4 are much higher than the other storms. Therefore, as revealed here, the second-order stationarity is not appropriate for describing the spatial structures of storms No. 1, 2, and 4, especially if their semi-variograms do not reach an asymptotic behavior, as in this case. However, some physical behaviors of the river basin, such as the range, which reflects topographic effects on rainfall distributions, can still be explained by use of spherical semi-variograms.

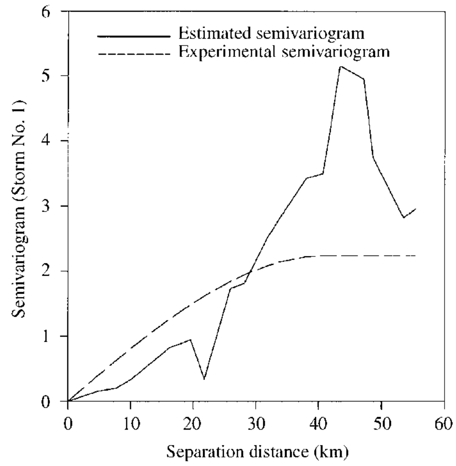

Fig. 3 Semivariogram of storm No. 1

Upon viewing Table 5, it is apparent that the ranges of the six storms are from 40~52 kilometers. This reveals that the structure of the rainfall distributions in the study area is highly correlated within this range. This result is very close to the downstream length, around 47-km of the Tsen-Wen River, which flows at the Chia-Nan plain. If an ensemble mean of the ranges with multi-realization is taken into account, the mean value is about 46.17-km. This value is almost equal to the downstream length, which reveals a direct correlation between land structure and rainfall distributions.

This argument can be explained by recalling that storm models for calculating possible precipitation have to consider topographic changes. This reveals that the storm structure is disturbed under orographic influences. Therefore, large-scale vertical movements of weather systems in mountainous areas often produce a complexity of spatial rainfall patterns. However, in this study, the semivariogram investigations give a result which reveals that topography with around 47-km length on a plain produces a spatially correlated rainfall distribution. It indicates that the rainfall distribution, assumed to be isotropically stationary, and statistically homogeneous is reasonable in order to decide the range of the highly correlated rainfall distribution.

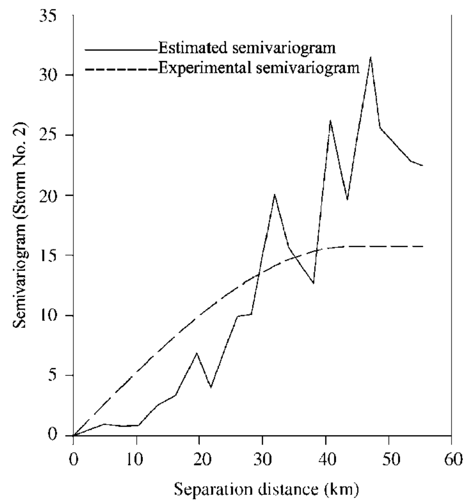

Fig. 4 Semivariogram of storm No. 2

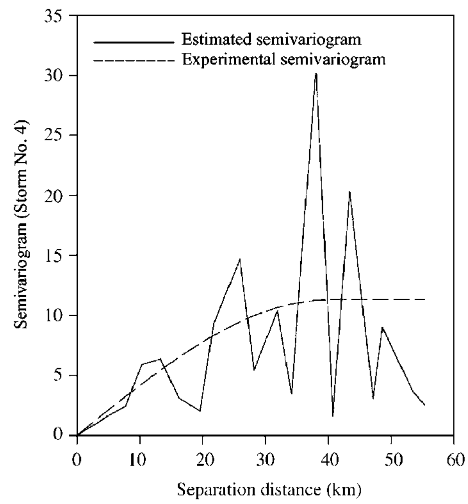

Fig. 6 Semivariogram of storm No. 4

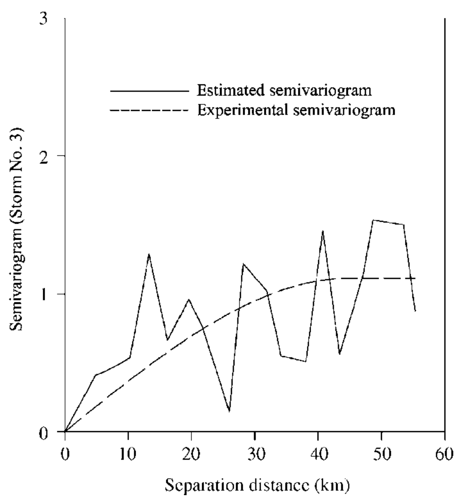

Fig. 5 Semivariogram of storm No. 3

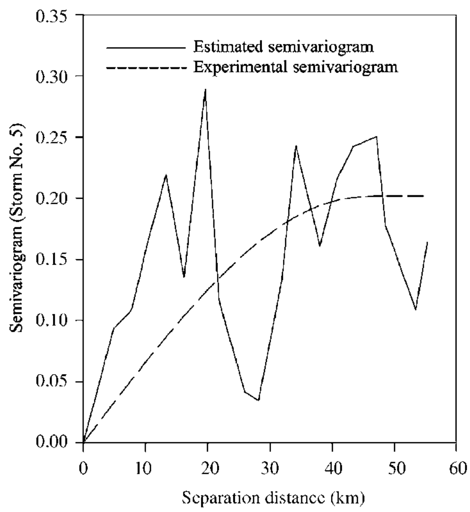

Fig. 7 Semivariogram of storm No. 5

2. Mean areal Precipitation Estimates of Rainfall Distributions

Using the spherical semivariogram, the kriging optimal weights of the 13 stations in Eq. (11) can be computed after a set of paired stations are uniformly selected from the study area. The weights for each storm obtained by the kriging method are listed in Table 6. A comparison is made between the weights obtained by use of the kriging method and those by the Thiessen method (Table 3). The comparison shows that those yielded by the kriging method for stations No. 2, 3, 4, 5, 6, and 13 (most of the stations in the eastern area of the study basin, except for station No. 13) are highly different from those by the Thiessen method. The difference is due to topographic effects from the mountainous area, thus creating a varied rainfall distribution. However, for stations No. 1, 7, 8, 9, 10, 11, and 12 (most of the stations in the western area of the study basin, except for station 1), the weights by the kriging method are similar to those by the Thiessen method. The indication from these weights is that topographic effects from the plain area create a more uniform rainfall distribution. This reveals that the topographic effects for the rainfall distribution in the study area should be investigated with the kriging method to obtain the estimates of the mean areal precipitation.

With the six sets of optimal weights, the estimates of mean areal precipitation of six storms are evaluated. Meanwhile, the standard deviations of the kriging error of estimation (a square root of Eq. (12)) are calculated. The estimated mean areal precipitation and standard deviations are listed in Table 7. For comparison, the mean areal precipitation estimated by the Thiessen method is also included in Table 7.

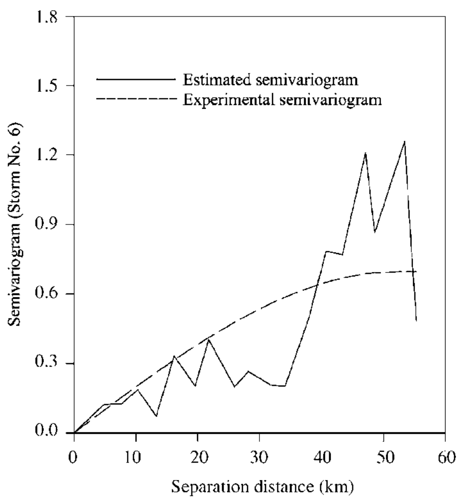

Fig. 8 Semivariogram of storm No. 6

It is worth noting that differences between these two methods for the six storms are less than 2.5%. In addition, all of the differences are less than the standard deviation of the kriging error of estimation. With the standard deviation of the kriging error of estimation, higher standard deviations can be found for storms No. 1, 2, and 4 than for storms No. 3, 5, and 6.

These results reveal two factors about the spatial structure of storms No. 1, 2, and 4. First, the spatial structures of the storms cannot follow the assumption of the isotropic stationary process due to their higher estimated variance. This finding is the same as the one previously stated under *Spatial structures of rainfall distributions* in this paper. Second, the higher standard deviations reflect topographic effects on the rainfall distributions from the mountainous area.

On the other hand, because of lower standard deviations, storms 3, 5, and 6, represented in Figs. 5, 7, and 8, present good results with the functions of the spherical semivariograms. Less variance exists between the experimental and spherical semi-variograms. Their lower standard deviations also reveal topographic effects on the rainfall distributions from the plain area.

V. CONCLUSION

The spatial structure and mean areal precipitation of rainfall in the Tsen-Wen River basin were investigated by the kriging method with semi-variograms. The correlating range (46.17-km) of the six storms was found to be very close to the downstream length (47-km) of the Tsen-Wen River in its flood plain. This reflects that it is possible to use the semivariograms of the six storms in order to investigate the impacts of topography on storm rainfall distributions in the Tsen-Wen River basin with the isotropic stationary assumption.

Table 5 The spherical semivariograms of six storms

Storm No.	Spherical Semivariogram
No.1	$\gamma = \begin{cases} 2.2439[1.5(\frac{h}{41}) - 0.5(\frac{h}{41})^3], & h \le 41 \\ 2.2439 & h > 41 \end{cases}$
No.2	$\gamma = \begin{cases} 15.7562[1.5(\frac{h}{44}) - 0.5(\frac{h}{44})^3], & h \le 44 \\ 15.7562 & h > 44 \end{cases}$
No.3	$\gamma = \begin{cases} 1.1147[1.5(\frac{h}{45}) - 0.5(\frac{h}{45})^3], & h \le 45 \\ 1.1147 & h > 45 \end{cases}$
No.4	$\gamma = \begin{cases} 11.309[1.5(\frac{h}{40}) - 0.5(\frac{h}{40})^3], & h \le 40 \\ 11.309 & h > 40 \end{cases}$
No.5	$\gamma = \begin{cases} 0.2018[1.5(\frac{h}{45}) - 0.5(\frac{h}{45})^3], & h \le 45 \\ 0.2018 & h > 45 \end{cases}$
No.6	$\gamma = \begin{cases} 0.6963[1.5(\frac{h}{52}) - 0.5(\frac{h}{52})^3], & h \le 52 \\ 0.6963 & h > 52 \end{cases}$

Analysis of the estimates of the mean areal precipitation indicates that these estimates resulting from the kriging method and the Thiessen method differ by less than 2.5%. Basically, the process of second-order stationarity for rainfall distributions is statistically isotropic. The process is similar to the uniformly distributed process, which is traditionally applied to obtain the estimate of the mean areal precipitation with the Thiessen method. However, a detailed examination of the rainfall distributions with the kriging method in the study area reveals that impacts of the topography on the rainfall distributions exist, which cannot be found with the Thiessen method.

It is worth noting that the smaller the standard deviation of the estimated error, as shown in this study, the more appropriate it is to use the assumption of the statistically isotropic and stationary

Table 6 List of optimal weights of the six storms obtained by the kriging method

Station No.	No.1	No.2	No.3	No.4	No.5	No.6
1	0.0865	0.0880	0.0880	0.0860	0.0880	0.0885
2	0.1179	0.1175	0.1175	0.1178	0.1175	0.1182
3	0.0512	0.0510	0.0513	0.0513	0.0513	0.0520
4	0.0791	0.0793	0.0794	0.0792	0.0794	0.0803
5	0.1323	0.1326	0.1326	0.1321	0.1326	0.1325
6	0.0519	0.0497	0.0494	0.0529	0.0494	0.0493
7	0.0585	0.0576	0.0575	0.0590	0.0575	0.0594
8	0.0517	0.0520	0.0515	0.0513	0.0515	0.0504
9	0.0772	0.0788	0.0791	0.0767	0.0791	0.0775
10	0.0955	0.0955	0.0957	0.0957	0.0957	0.0976
11	0.0394	0.0399	0.0406	0.0396	0.0406	0.0408
12	0.0697	0.0688	0.0682	0.0696	0.0682	0.0650
13	0.0891	0.4243	0.0893	0.0887	0.0893	0.0886
Sum	1.0000	1.0000	1.0001	0.9999	1.0001	1.0001

Table 7 Mean areal precipitation (MAP) estimates obtained by both the kriging and the Thiessen methods and their standard deviations

Storm No.	MAP estimate, Z^*		Comparison of MAP		Standard Deviation of Estimator Error
	Kriging Z_1^*	Thiessen Z_2^*	$Z_1^*-Z_2^*$	$(Z_1^*-Z_2^*)/(Z_2^*)$ ^{*100}	
1	28.33	28.08	0.25	0.88	1.16
2	56.35	55.23	1.12	2.03	2.97
3	28.33	28.69	-0.36	-1.25	0.78
4	41.80	42.31	-0.51	-1.21	2.64
5	19.20	19.65	-0.45	-2.29	0.33
6	13.50	13.60	-0.10	-0.74	0.57

processes for investigating the storm structures. This is a very important result for rainfall models using the kriging method on an island that has been ignored in the past. From the study, it can be generally agreed that the kriging method used with semivariograms is possible for the investigation of the spatial structure of rainfall distribution and the estimation of mean areal precipitation on this oceanic island.

ACKNOWLEDGMENTS

National Science Council under grants NSC 87-2211-E-224-017 and NSC-84-2211-E-033-004 sponsors this work. The author would like to acknowledge Ms. Resta Saphore-Cheng who helps in writing. Also, the cooperation of Mr. I.-F. Lu and Miss C.-D. Cheng, who are working in the research center of soil & water resources and natural disastrous prevention, help in creating the figures.

NOMENCLATURE

a	the range
C_o	nugget effect
C_o+C	sill
$E[Z(\underline{u}_1)\ Z(\underline{u}_2)]$	covariance function of rainfall distribution betweem $\underline{u}_1$ and $\underline{u}_2$
$\lvert\underline{h}\rvert$	separation distance of paired stations
m	mean areal precipitation
$n_{\underline{h}}$	number of paired stations within each interval defined by $\underline{h}_1$ and $\underline{h}_2$
S_0	mean areal precipitation over some area of interest
$Z(\underline{u}_i)$	measured rainfall at observation points $\underline{ui}$
$Z^*_{S_O}$	best linear unbiased estimate of the true unknown value, Z_{S_O}
$\gamma(\underline{h})$	experimental semivariogram of rainfall distributions $Z(\underline{u}_i)$ and is a function of $\underline{h}$ only or spherical semivariogram
$\gamma(\underline{h}\,)$	estimated semivariogram of rainfall distributions from discrete data observed at the stations
λ_I	optimal weight at location $\underline{u}_i$

μ Lagrangian multiplier
σ^2 variance of rainfall distribution

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Manuscript Received: Oct. 21, 2000
Revision Received: Feb. 28, 2001
and Accepted: Mar. 13, 2001

曾文溪流域降雨分佈之平均雨量及空間研究結構

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摘 要

本研究以序率統計中克利金法進行降雨空間結構研究。其中以半變異元原理檢定於曾文溪流域發生的六場颱風降雨之空間結構及其平均降雨量。

研究發現曾文溪流域的地貌變化會對流域內降雨分佈造成影響。此一影響可以由克利金法計算所得之平均降雨量分佈不同於傳統徐昇法計算所得之平均降雨量，此一差量約爲 2.5% 的差量。而另一證據爲克利金法所求得之六場颱風降雨分佈的平均範圍（range）約爲46.17公里，此一結果與曾文溪河川流入平原地區後的總長度 47 公里非常接近。此外，本研究亦發現，當降雨分佈的標準偏差愈小時，本研究所假設之二階定常檢定降雨空間結構假設越合適。

關鍵詞：徐昇法，克利金法，序率統計，半變異元。