

Performance evaluation of random power capture on mobile communications[†]

Jyh-Horng Wen^{1,*}, Kuo-Gen Hsu¹, Jet-Chau Wen² and Yi-Show Chen¹

¹*Institute of Electrical Engineering, National Chung Cheng University, Chiayi, Taiwan*

²*Department of Environmental and Safety Engineering, National Yunlin University of Science and Technology, Yun-lin, Taiwan*

SUMMARY

Power assignment schemes are man-made methods to enhance the capture effect of radio communications. In a previous study, Wen and Yang investigated the combined capture effect of the fixed power assignment scheme, Rayleigh fading, and near-far effect on the performance of packet radios. The performance analysis was limited to an infinite population environment. This assumption is reasonable for a conventional packet radio system with a vast service area. However, for a cellular mobile system, a finite population model should be used. In this paper, we analyse the combined *natural* and *man-made* capture effect on the performance of a cellular system with finite population in each cell. A random power assignment scheme is adopted to produce the man-made capture. The system throughput and delay are carried out by a Markov model. Some numerical calculations are used to demonstrate the degree of performance improvement. Copyright © 2001 John Wiley & Sons, Ltd.

KEY WORDS: random power assignment scheme; capture effect; ALOHA protocol; throughput; delay

1. INTRODUCTION

Since Abramson [1] introduced the ALOHA protocol in 1970, the research of random access techniques has become one of the major subjects in packet radio communication systems. Numerous papers [2–18] have engaged in the improvement of throughput on the ALOHA-based system owing to its attractiveness in simplicity and reasonable efficiency.

The performance improvement of ALOHA system owing to capture effect has been studied in many papers. Some fading factors, e.g. Rayleigh fading, near/far effect and shadow fading, are considered to enhance the capture effect. Zainal and Garcia [3] studied the individual effect of

*Correspondence to: Jyh-Horng Wen, Institute of Electrical Engineering, National Chung Cheng University, Ming Hsiung, Chiayi, Taiwan.

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Rayleigh fading while Namislo [4] and Goodman and Saleh [5] discussed the effect of near/far phenomenon (path-loss) only. The combined impact of Rayleigh fading and near/far effect was investigated in References [6,7]. The shadow fading was studied in References [8,9] and the combined impact of Rayleigh fading, shadow fading, and near/far effect was discussed in References [10,11]. All of those studies dealt with how 'natural' capture effect is related to the performance of ALOHA protocol.

There exists a *man-made* method to enhance the capture effect. Metzner [12] initially proposed to divide the users into two groups, one transmitting at high-power level and the other at low-power level. As a result, he showed that the maximum throughput of a slotted ALOHA communication system can be increased from 36.8 per cent to around 53 per cent. Shacham [13] studied a more general case that each packet is assigned one of k power levels, and he used a perfect capture model to analyse the resulting throughput-delay performance. Cidon and Sidi [14] adopted the same strategy and assumption in the performance analysis of the tree collision resolution algorithm (CRA) [15,16]. Recently, Cidon *et al.* [17] and Lee [18] proposed the random power assignment scheme to analyze the performance of CRA and slotted ALOHA protocols, respectively. Instead of dividing the users into priority groups, every user was assigned an identical probabilistic rule and an identical set of power levels. This random power assignment scheme can eliminate the priority problem caused by the fixed power assignment scheme.

While considering the mobile radio networks, two unrealistic assumptions in References [12,14], [17,18] should be pointed out. First, all the proposed capture models of those papers concentrate only on the effect of different man-made power level. They do not incorporate the natural capture effect in their analyses. However, in the mobile radio environment, nature fading always exists and cannot be neglected. Second, 'perfect' capture ability had been adopted by those papers. This assumption is apparently too optimistic because well-designed FM receivers only have a capture ratio approximately equal to $\sqrt{2}$ [19]. In Reference [20], Wen and Yang have studied the performance of two different power assignment schemes in packet radio networks over Rayleigh fading channel. In addition, the combined effect of fixed power assignment scheme, Rayleigh fading, and near-far effect on the performance of slotted ALOHA was further studied in Reference [21]. They also took account of a general capture model with every value of capture ratio. Although Wen and Yang [20,21] considered a realistic model, the performance analysis was limited to an infinite population environment. This assumption is reasonable for a conventional packet radio system with a vast service area. Nevertheless, with a cellular mobile system under consideration, a finite population model should be used. Herein, we analyse the combined natural and man-made capture effect on the performance of a cellular system with finite population in each cell. The overall impact of Rayleigh fading and near/far effect will be considered.

The rest of this paper is organized as follows. Section 2 gives a description for the system model. Section 3 analyses the system performance based on a Markov model. Numerical results and discussions are given in Section 4. Finally, concluding remarks are summarized in Section 5.

2. SYSTEM MODEL

We consider a cellular system, where finite users exist in each cell and share a common channel to communicate with a central base station. The accessing protocol is slotted ALOHA with an acknowledgement transmitted by the base station whenever a packet is successfully received.

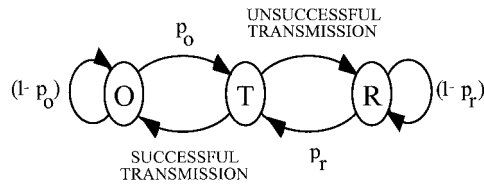


Figure 1. State machine for an individual user.

Herein, we assume an ideal system where acknowledgements are never lost [4]. Furthermore, we assume that the signal-to-noise ratio at the receiver is high enough so that packets are never lost owing to noise. In other words, the interference of AWGN is neglected in this paper. Thus, an unsuccessful reception of a packet can only be caused by a collision with other packets [4,7,22,23,26].

We assume that N identical and mutually independent users exist in each cell. The state machine of every user has three modes — origination (O), transmission (T), and retransmission (R), as shown in Figure 1. Transitions occur only at the beginning (or end) of a time slot. A user in the origination state (O) generates a packet with probability p_0 . When a packet is generated, the user goes into the transmission state (T) for one slot interval. At the end of the slot, if the transmission was successful (as indicated by an immediate acknowledge) the user returns to the origination state; otherwise, the user moves to the retransmission state (R), where, with retransmission probability p_r , the user keeps trying to transmit the packet (by going back to state T for one time slot) until it is successfully received. The user then returns to the origination state and starts generating new packets. The behaviours of all users' state machines are identical and mutually independent [4,17].

Since, in mobile radio environments, multipath fading always exists and cannot be ignored, we assume that each transmitted packet arrives at the base station with independent Rayleigh fading. Thus the received r.m.s amplitude A of a packet is a random variable with probability density function (p.d.f.) [7,10,24,25]

$$P(A) = \frac{2A}{\mu} e^{-A^2/\mu}, \quad A \geq 0 \quad (1)$$

where $\mu = \overline{A^2}$ is the mean received power level. We further assume that we are working with a slow varying fading channel so that the power level of a received packet remains approximately constant during the interval of its reception [7,10].

In conventional analysis of slotted ALOHA protocol, it is generally assumed that packets involved in a collision are destroyed [1]. However, in mobile radio networks, the packets usually experience fading, which results in those packets having different power levels at the receiver and makes capture effect possible [2–11]. Restated, if a collision occurs, the receiver may recover one packet involved in the collision. In this paper, we use the power advantage (or signal-to-interference ratio advantage) measure and define the capture criterion as follows. Suppose a total of $i + 1$ packets collide and let W_0, W_1, \dots, W_i be the power levels of the packets involved in the collision, then the packet with power level W_0 succeeds if $W_0/(W_1 + \dots + W_i) > \rho$. The threshold

ρ , which is no less than one, is the so-called capture ratio. Notice that $\rho = 1$ corresponds to perfect capture, i.e. the desired packet succeeds as long as its power level is stronger than the sum of those of the other packets participating in the collision. This type of capture effect is also investigated in References [4,6,7,20].

In order to inject the man-made capture effect, random power assignment scheme is considered in this paper. In random power assignment scheme [12,17,18], whenever a user has a packet to transmit or retransmit it randomly chooses one of L different power levels (numbered $1, 2, \dots, L$). Let μ_i , $1 \leq i \leq L$, be the mean received power level of the packet using the i th level. We assume $\mu_1 \geq \mu_2 \geq \dots \geq \mu_L$. In other words, if $i < j$, a level- i packet has a higher transmitting power level than a level- j packet (herein, we temporarily neglect the path-loss effect). Since we have assumed a Rayleigh fading channel, the received power W_i of a packet using the i th transmitting power level forms an exponential distributed random variable with p.d.f. [7,10,24,25]

$$f_{W_i}(x) = \frac{1}{\mu_i} e^{-x/\mu_i}, \quad x \geq 0 \quad (2)$$

However, when considering the near/far effect, the mean received power μ_i is proportional to $r^{-\alpha}$, where r denotes the distance from the transmitting user to the base station and α is the propagation constant. In free space $\alpha = 2$, while in a mobile radio system we have $2 \leq \alpha \leq 5$ [27,28]. For convenience, we use $\alpha = 4$ throughout this paper.

3. ANALYSES OF SYSTEM THROUGHPUT AND DELAY

Based on the assumption of finite population, we can use the Markov model presented in Reference [4] to derive the system performance for each cell. Since the major damage on system performance arises from a collision with intra-cell packets, for simplicity, the co-channel interference and adjacent channel interference are ignored in this paper. We define the state variable as the number of terminals being in the retransmission state. Then, the one-step transition probability $\pi_{n,m}$ from state n to m can be obtained as follows [4]:

$$\pi_{n,m} = \begin{cases} 0 & \text{if } m < n - 1 \\ (1 - p_0)^{N-n} \sum_{i=1}^n \binom{n}{i} p_r^i (1 - p_r)^{n-i} q_i & \text{if } m = n - 1 \\ \left\{ \binom{N-n}{N-m} (1 - p_0)^{N-m-1} p_0^{m-n} \sum_{i=0}^n \binom{n}{i} (1 - p_r)^{n-i} p_r^i \right. \\ \quad \left. \times \left[(1 - p_0)(1 - q_{i+m-n}) + \frac{N-m}{m-n+1} p_0 q_{i+m-n+1} \right] \right\} & \text{if } m \geq n \end{cases} \quad (3)$$

Note that the transition probability depends on the capture probability q_i , which is defined as the probability that one out of i colliding packets is correctly received. Obviously, if we can obtain the capture probability q_i , the transition probability $\pi_{n,m}$ can be calculated from Equation (3) and the steady state probability π_n ($0 \leq n \leq N$) of the system being in state n can be obtained from the

finite set of linear equations

$$\begin{aligned} \boldsymbol{\pi} &= \boldsymbol{\pi} \mathbf{P} \\ \sum_{n=0}^N \pi_n &= 1 \end{aligned} \tag{4}$$

where $\mathbf{P} = [\pi_{n,m}]_{(N+1) \times (N+1)}$, $\boldsymbol{\pi} = [\pi_n]_{1 \times (N+1)}$. Since the expected throughput of the network in state n can be found to be

$$\begin{aligned} f_n &= \sum_{j=0}^{N-n} \binom{N-n}{j} p_0^j (1-p_0)^{N-n-j} \\ &\quad \times \sum_{i=0}^n \binom{n}{i} p_r^i (1-p_r)^{n-i} q_{i+j} \end{aligned} \tag{5}$$

with the knowledge of π_n ($0 \leq n \leq N$), we can calculate the overall throughput via

$$f = \sum_{i=0}^N \pi_i f_i \tag{6}$$

and the average state (expected backlog) by

$$S = \sum_{i=0}^N \pi_i i \tag{7}$$

Finally, the expected value of the delay in packets can be expressed by the expected backlog divided by the average throughput, that is

$$D = \frac{S}{f} = \frac{\sum_{i=0}^N \pi_i i}{f} \tag{8}$$

The remaining work for us is to derive the capture probability, q_i , for different fading environments.

3.1. The capture probability q_i under Rayleigh fading only

For convenience, let M_i , $1 \leq i \leq L$, denote the random variable that represents the number of packets using the i th power level participating in a collision. We use W_{ij} , $1 \leq i \leq L$, $1 \leq j \leq M_i$ to represent the power level of the j th packet among the M_i packets mentioned above. Obviously, q_i can be obtained as follows:

$$q_i = \sum_{h_1=0}^i \sum_{h_2=0}^{i-h_1} \sum_{h_3=0}^{i-h_1-h_2} \cdots \sum_{h_{L-1}=0}^{i-h_1-\cdots-h_{L-2}} \binom{i}{h_1, h_2, \dots, h_L} \left(\frac{1}{L}\right)^{h_1} \left(\frac{1}{L}\right)^{h_2} \cdots \left(\frac{1}{L}\right)^{h_L}$$

$$\begin{aligned}
 & \times \left\{ \binom{h_1}{1} P \left[\frac{W_{10}}{\sum_{j_1=1}^{h_1-1} W_{1j_1} + \sum_{j_2=1}^{h_2} W_{2j_2} + \dots + \sum_{j_L=1}^{h_L} W_{Lj_L}} > \rho \right] \right. \\
 & + \binom{h_2}{1} P \left[\frac{W_{20}}{\sum_{j_1=1}^{h_1} W_{1j_1} + \sum_{j_2=1}^{h_2-1} W_{2j_2} + \dots + \sum_{j_L=1}^{h_L} W_{Lj_L}} > \rho \right] \\
 & \vdots \\
 & \left. + \binom{h_L}{1} P \left[\frac{W_{L0}}{\sum_{j_1=1}^{h_1} W_{1j_1} + \sum_{j_2=1}^{h_2} W_{2j_2} + \dots + \sum_{j_L=1}^{h_L-1} W_{Lj_L}} > \rho \right] \right\} \tag{9}
 \end{aligned}$$

where h_m , $1 \leq m \leq L$, denotes the number of packets using level- m power among the i colliding packets, and W_{m0} , $1 \leq m \leq L$, indicates the power level of a desired packet when it uses level- m power. Note that $h_1 + h_2 + \dots + h_L = i$. Let

$$Z_m = \sum_{j_m}^{h_m} W_{m,j_m}, \quad 1 \leq m \leq L \tag{10}$$

Since the slow Rayleigh fading channel is assumed in this paper, the power level of each packet forms i.i.d. exponential distributed random variable with p.d.f. as shown in Equation (2), and then Z_m has Gamma distribution with p.d.f. $\Gamma(h_m, 1/\mu_m)$. In Reference [20], Wen and Yang have derived that

$$\begin{aligned}
 & P \left[\frac{W_{i0}}{\sum_{j_1=1}^{h_1} W_{1j_1} + \sum_{j_2=1}^{h_2} W_{2j_2} + \dots + \sum_{j_L=1}^{h_L} W_{Lj_L}} > \rho \right] \\
 & = P \left[\frac{W_{i0}}{Z_1 + Z_2 + \dots + Z_L} > \rho \right] \\
 & = \left(\frac{1}{1 + \rho\mu_1/\mu_i} \right)^{h_1} \left(\frac{1}{1 + \rho\mu_2/\mu_i} \right)^{h_2} \dots \left(\frac{1}{1 + \rho\mu_L/\mu_i} \right)^{h_L} \tag{11}
 \end{aligned}$$

Therefore, it yields

$$\begin{aligned}
 q_i & = \sum_{h_1=0}^i \sum_{h_2=0}^{i-h_1} \sum_{h_3=0}^{i-h_1-h_2} \dots \sum_{h_{L-1}=0}^{i-h_1-\dots-h_{L-2}} \binom{i}{h_1, h_2, \dots, h_L} \left(\frac{1}{L} \right)^{h_1} \left(\frac{1}{L} \right)^{h_2} \dots \left(\frac{1}{L} \right)^{h_L} \\
 & \times \left\{ \binom{h_1}{1} \left(\frac{1}{1 + \rho\mu_1/\mu_1} \right)^{h_1-1} \left(\frac{1}{1 + \rho\mu_2/\mu_1} \right)^{h_2} \dots \left(\frac{1}{1 + \rho\mu_L/\mu_1} \right)^{h_L} \right\}
 \end{aligned}$$

$$\begin{aligned}
& + \binom{h_2}{1} \left(\frac{1}{1 + \rho \mu_1 / \mu_2} \right)^{h_1} \left(\frac{1}{1 + \rho \mu_2 / \mu_2} \right)^{h_2 - 1} \cdots \left(\frac{1}{1 + \rho \mu_L / \mu_2} \right)^{h_L} \\
& \vdots \\
& + \binom{h_L}{1} \left(\frac{1}{1 + \rho \mu_1 / \mu_L} \right)^{h_1} \left(\frac{1}{1 + \rho \mu_2 / \mu_L} \right)^{h_2} \cdots \left(\frac{1}{1 + \rho \mu_L / \mu_L} \right)^{h_L - 1} \} \quad (12)
\end{aligned}$$

After mathematical manipulation, it yields

$$q_i = \frac{i}{L} \sum_{m=1}^L \left[\sum_{n=1}^L \frac{1}{L(1 + \rho \mu_n / \mu_m)} \right]^{i-1} = \frac{i}{L} \sum_{m=1}^L \left[\sum_{n=1}^L \frac{1}{L(1 + \rho k^{m-n})} \right]^{i-1} \quad (13)$$

here we assume that $k = \mu_1 / \mu_2 = \mu_2 / \mu_3 = \cdots = \mu_{L-1} / \mu_L$. In Equation (13), the results for $q_0 = 0$ and $q_1 = 1$ are expected.

3.2. The capture probability q_i under Rayleigh fading and near/far effect

In this sub-section, we incorporate the near/far effect into consideration. For convenience, we assume that N users uniformly distribute in a service circle of radius R . Without loss of generality, we can assume that $R = 1$ (which is equivalent to normalizing all distance by R). The normalized distance r of a user from the base station is a random variable with probability density function

$$f(r) = \begin{cases} 2r, & 0 \leq r \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad (14)$$

Since we consider the Rayleigh fading and near/far effect, the power level of a packet, W , with its transmitter at a fixed distance r from the receiver, has an exponential distribution with p.d.f.

$$f_{w_i}(x|r) = \frac{1}{\mu_i / r^4} e^{-x / (\mu_i / r^4)}, \quad x \geq 0, \quad 0 < r \leq 1 \quad (15)$$

Equation (15) implies the average received power level of a packet using level i is proportional to r^{-4} .

According to Equation (11), a desired packet using level- m power in contending with the other one using level- n power has successful probability

$$P_{mn} = \left(\frac{1}{1 + \rho \mu_n / \mu_m} \right) \quad (16)$$

However, with the near/far effect under consideration, the mean received power level of a packet transmitted at a distance r from the base station using level i is proportional to r^{-4} .

Therefore, Equation (16) should be rewritten as

$$P_{mn}(r, s) = \left(\frac{1}{1 + \rho(\mu_n/\mu_m)(r/s)^4} \right), \quad 0 < r, s \leq 1 \quad (17)$$

where r and s represent the distances of the desired-packet user and the interfering-packet user from the base station, respectively. Averaging $P_{mn}(r, s)$ over the entire range of s , we obtain

$$\begin{aligned} P_{mn}(r) &= \int_0^1 P_{mn}(r, s) f(s) ds \\ &= \int_0^1 \frac{2s ds}{1 + \rho(\mu_n/\mu_m)(r/s)^4} \\ &= 1 - (\rho k^{m-n})^{1/2} r^2 \tan^{-1} \left[\frac{1}{(\rho k^{m-n})^{1/2} r^2} \right] \end{aligned} \quad (18)$$

In fact, $P_{mn}(r)$ denotes the successful probability of a desired packet transmitted with level- m power at a fixed distance r from the base station in the presence of an interfering packet using level- n power.

Next, the capture probability for a desired packet transmitted at a fixed distance r from the base station under the condition of $(i-1)$ interfering packets transmitted at distances S_j , $1 \leq j \leq i-1$, from the base station can be obtained via

$$\begin{aligned} q_i(r, S_j; 1 \leq j \leq i-1) &= \sum_{h_1=0}^i \sum_{h_2=0}^{i-h_1} \sum_{h_3=0}^{i-h_1-h_2} \cdots \sum_{h_{i-1}=0}^{i-h_1-\cdots-h_{i-2}} \binom{i}{h_1, h_2, \dots, h_L} \left(\frac{1}{L}\right)^{h_1} \left(\frac{1}{L}\right)^{h_2} \cdots \left(\frac{1}{L}\right)^{h_L} \\ &\quad \times \left\{ \binom{h_1}{1} P_{s_1}(r, s_{1,i_1}, s_{2,i_2}, \dots, s_{L,i_L}, 1 \leq i_1 \leq h_1 - 1, \right. \\ &\quad \quad \quad \left. 1 \leq i_2 \leq h_2, \dots, 1 \leq i_L \leq h_L | M_1 = h_1, M_2 = h_2, \dots, M_L = h_L) \right. \\ &\quad \quad \quad \left. + \binom{h_2}{1} P_{s_2}(r, s_{1,i_1}, s_{2,i_2}, \dots, s_{L,i_L}, \right. \\ &\quad \quad \quad \left. 1 \leq i_1 \leq h_1, 1 \leq i_2 \leq h_2 - 1, \dots, 1 \leq i_L \leq h_L | M_1 = h_1, M_2 = h_2, \dots, M_L = h_L) \right. \\ &\quad \quad \quad \vdots \\ &\quad \quad \quad \left. + \binom{h_L}{1} P_{s_L}(r, s_{1,i_1}, s_{2,i_2}, \dots, s_{L,i_L}, 1 \leq i_1 \leq h_1, 1 \leq i_2 \leq h_2, \dots, 1 \leq i_L \right. \\ &\quad \quad \quad \left. \leq h_L - 1 | M_1 = h_1, M_2 = h_2, \dots, M_L = h_L) \right\} \end{aligned} \quad (19)$$

where s_{n,i_n} , $1 \leq n \leq L$, denotes the distance from the transmitter of the i_n th interfering packet among M_n packets using level- n power to the base station. In Equation (19), the term P_{s_c} , $1 \leq c \leq L$, denotes the successful probability of a desired packet using level- c power at a fixed distance r from the base station under the condition that $M_1 = h_1, M_2 = h_2, \dots, M_L = h_L$ packets participating in a collision.

Similar to Equation (11), P_{s_c} can be derived as follows:

$$\begin{aligned}
 & P_{s_c}(r, s_{1,i_1}, s_{2,i_2}, \dots, s_{L,i_L}, 1 \leq i_1 \leq h_1, \dots, 1 \leq i_{c-1} \leq h_{c-1}, 1 \leq i_c \leq h_c - 1, 1 \leq i_{c+1} \leq h_{c+1}, \\
 & \dots, 1 \leq i_L \leq h_L | M_1 = h_1, M_2 = h_2, \dots, M_L = h_L) \\
 &= \prod_{i_1=1}^{h_1} \left(\frac{1}{1 + \rho(\mu_1/\mu_c)(r/s_{1,i_1})^4} \right) \cdot \prod_{i_2=1}^{h_2} \left(\frac{1}{1 + \rho(\mu_2/\mu_c)(r/s_{2,i_2})^4} \right) \cdots \prod_{i_c=1}^{h_c-1} \left(\frac{1}{1 + \rho(\mu_c/\mu_c)(r/s_{c,i_c})^4} \right) \\
 & \cdots \prod_{i_L=1}^{h_L} \left(\frac{1}{1 + \rho(\mu_L/\mu_c)(r/s_{L,i_L})^4} \right), \quad 1 \leq c \leq L \tag{20}
 \end{aligned}$$

Averaging Equation (20) over all possible locations of the interfering packets and using Equation (18), we obtain

$$P_{s_c}(r | M_1 = h_1, M_2 = h_2, \dots, M_L = h_L) = [P_{c1}(r)]^{h_1} [P_{c2}(r)]^{h_2} \cdots [P_{cc}(r)]^{h_c-1} \cdots [P_{cL}(r)]^{h_L}, \quad 1 \leq c \leq L \tag{21}$$

Therefore, by averaging Equation (19) over all possible locations of the interfering packets and using Equation (21), the average successful probability of one out of i colliding packets with the desired packet at a fixed distance r from the base station can be obtained as follows:

$$\begin{aligned}
 q_i(r) &= \sum_{h_1=0}^i \sum_{h_2=0}^{i-h_1} \cdots \sum_{h_{L-1}=0}^{i-h_1-\dots-h_{L-2}} \binom{i}{h_1, h_2, \dots, h_L} \left(\frac{1}{L}\right)^{h_1} \left(\frac{1}{L}\right)^{h_2} \cdots \left(\frac{1}{L}\right)^{h_L} \\
 & \times \left\{ \binom{h_1}{1} [P_{11}(r)]^{h_1-1} [P_{12}(r)]^{h_2} \cdots [P_{1L}(r)]^{h_L} \right. \\
 & + \binom{h_2}{1} [P_{21}(r)]^{h_1} [P_{22}(r)]^{h_2-1} \cdots [P_{2L}(r)]^{h_L} \\
 & \vdots \\
 & \left. + \binom{h_L}{1} [P_{L1}(r)]^{h_1} [P_{L2}(r)]^{h_2} \cdots [P_{LL}(r)]^{h_L-1} \right\} \tag{22}
 \end{aligned}$$

After mathematical manipulation, it becomes

$$\begin{aligned}
 q_i(r) &= \frac{i}{L} \left[\sum_{n=1}^L \frac{P_{1n}(r)}{L} \right]^{i-1} + \frac{i}{L} \left[\sum_{n=1}^L \frac{P_{2n}(r)}{L} \right]^{i-1} + \cdots + \frac{i}{L} \left[\sum_{n=1}^L \frac{P_{Ln}(r)}{L} \right]^{i-1} \\
 &= \frac{i}{L} \sum_{m=1}^L \left[\sum_{n=1}^L \frac{P_{mn}(r)}{L} \right]^{i-1} \tag{23}
 \end{aligned}$$

Finally, by averaging Equation (23) over all possible locations of $0 < r \leq 1$, the average successful probability of one out of i colliding packets can be obtained as

$$\begin{aligned}
 q_i &= \int_0^1 q_i(r) 2r \, dr \\
 &= \int_0^1 2r \frac{i}{L} \sum_{m=1}^L \left[\sum_{n=1}^L \frac{P_{mn}(r)}{L} \right]^{i-1} dr
 \end{aligned} \tag{24}$$

Note that, $q_0 = 0$ and $q_1 = 1$ can also be derived from Equation (24).

4. NUMERICAL RESULTS AND DISCUSSIONS

Throughout the numerical calculations, we use $N = 50$ and $\rho = 2$ for the number of subscribers in each cell and capture ratio. Figure 2(a) gives the capture probability q_i versus the state i for several values of k under random power assignment and Rayleigh fading with two power levels. We observe from this figure that q_i is improved as k is increased. This is owing to the fact that a larger k results in a stronger capture at the receiver. Figure 2(b) depicts the capture probability q_i versus the state i for different power levels under random power assignment and Rayleigh fading with $k = 2$. With the same reason for different k , we observe that q_i is improved as the number of power levels L is increased.

Figure 3(a) and (b) are plotted for the same parameters of Figure 2(a) and (b), respectively under random power assignment, Rayleigh fading, and near/far effect. Since $\alpha = 4$ is a typical propagation power law adopted in most of the papers regarding the near/far effect [5,7,9,10], the numerical results in these figures are calculated with $\alpha = 4$. From these figures, similar results can be obtained as k or L is increased. However, when compared to the case without the near/far effect, the saturated capture probability has been increased significantly because of the near/far

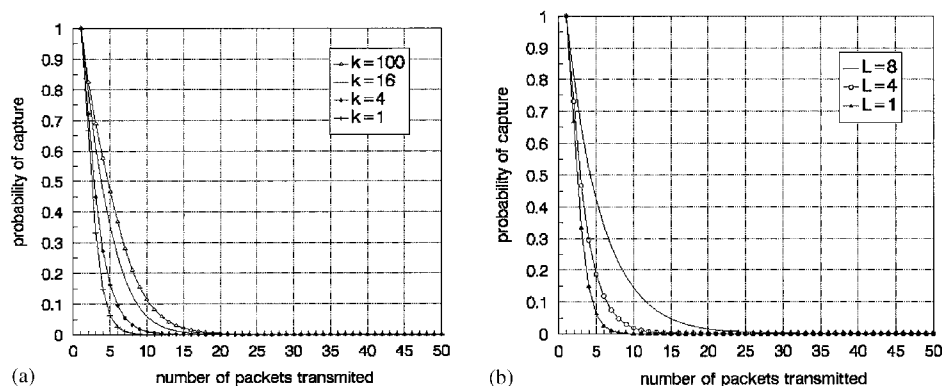


Figure 2. (a) q_i vs i under random power assignment and Rayleigh fading with $L = 2$; (b) q_i vs i under random power assignment and Rayleigh fading with $k = 2$.

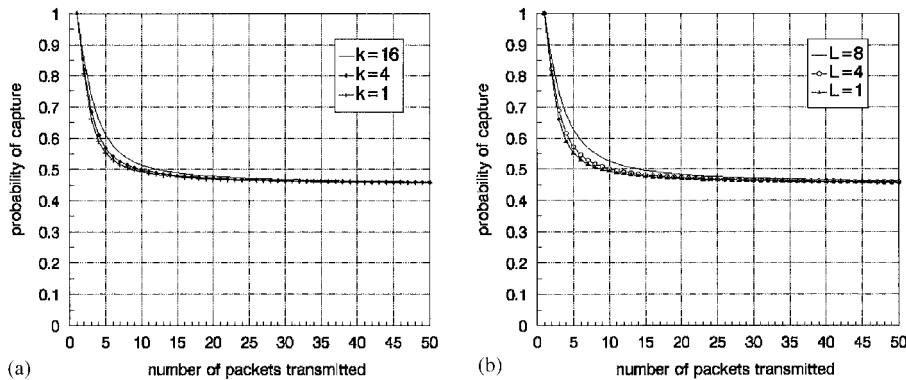


Figure 3. (a) q_i vs i under random power assignment, Rayleigh fading and near/far effect with $L = 2$; (b) q_i vs i under random power assignment, Rayleigh fading and near/far effect with $k = 2$.

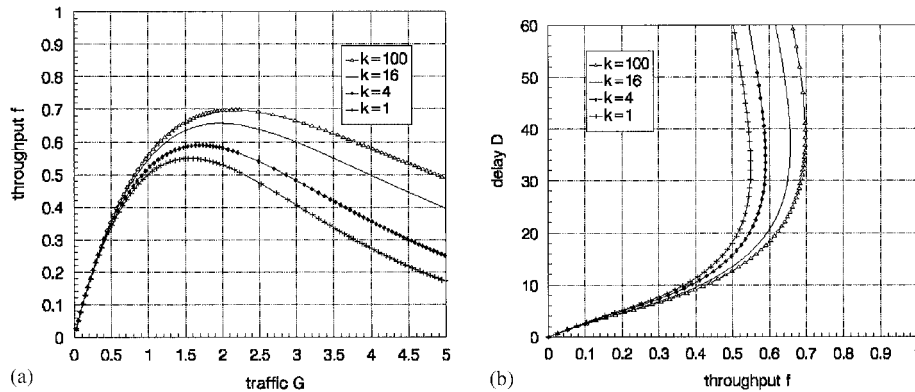


Figure 4. (a) f vs G under random power assignment and Rayleigh fading with $L = 2$; (b) D vs f under random power assignment and Rayleigh fading with $L = 2$.

effect. This finding implies that a large α enhances the near/far effect, consequently causing a better capture phenomenon, which is reasonable.

Figures 4 and 5 depict system performances without near/far effect while those with near/far effect are depicted in Figures 6 and 7. The system traffic G is calculated via $G = (N - S)p_0 + Sp_r$ with $p_r = 0.06$ and various p_0 . Figure 4(a) and (b), respectively show the system throughput and delay performances for several values of k under random power assignment and Rayleigh fading with $L = 2$. In these figures, the effect of power assignment scheme can be illustrated by the difference among various k values, while the enhancement from conventional slotted ALOHA to $k = 1$ is caused by the Rayleigh fading. It can be seen that both random power assignment and Rayleigh fading give a remarkable improvement in terms of system throughput and delay.

The effect of the number of power levels on the system throughput and delay with $k = 2$ are presented in Figure 5(a) and (b), respectively. Similar to Figure 4(a) and (b), a noticeable improvement in both system throughput and delay can be achieved if L is increased.

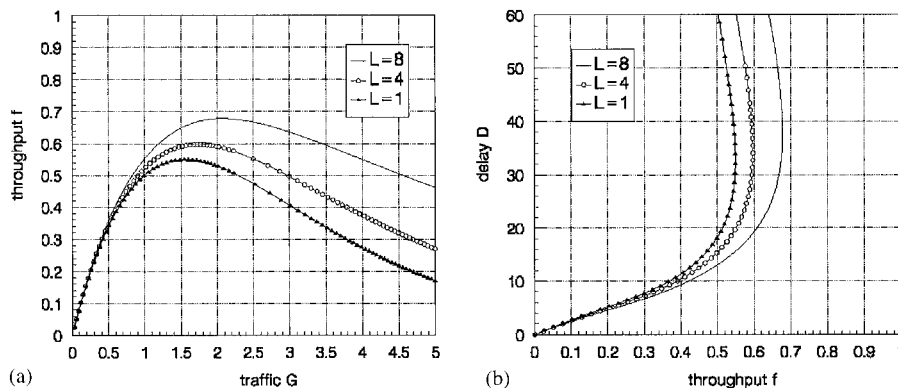


Figure 5. (a) f vs G under random power assignment and Rayleigh fading with $k = 2$; (b) D vs f under random power assignment and Rayleigh fading with $k = 2$.

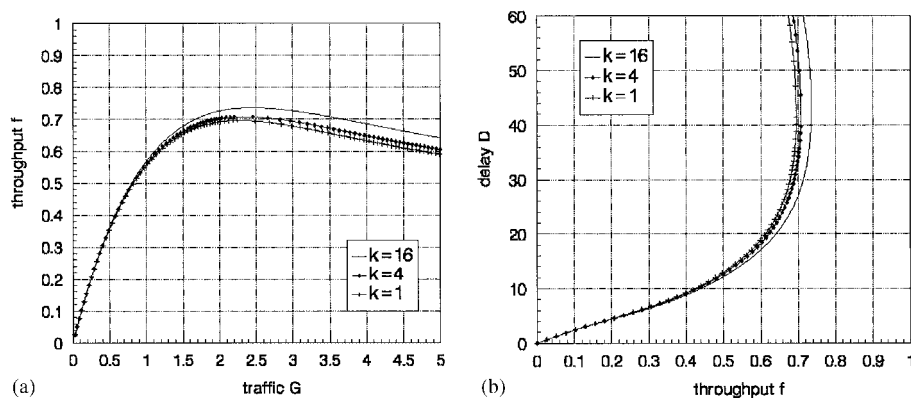


Figure 6. (a) f vs G under random power assignment, Rayleigh fading, and near/far effect with $L = 2$; (b) D vs f under random power assignment, Rayleigh fading, and near/far effect with $L = 2$.

Figure 6(a) and (b), respectively, show the system throughput and delay performances for several values of k under random power assignment, Rayleigh fading and near/far effect with $L = 2$. Note that the performance with $k = 1$ indicates the slotted ALOHA system without power assignment scheme. Therefore, the difference between $k = 1$ and any other curves in Figure 6(a) and (b) arises from the effect of random power assignment scheme. Comparing Figure 6(a) (or Figure 6(b)) with Figure 4(a) (or Figure 4(b)) clearly reveals that random power assignment scheme provides more significance in performance improvement if near/far effect disappears. Besides, the difference between the curve with $k = 1$ on Figure 6(a) (or Figure 6(b)) and that on Figure 4(a) (or Figure 4(b)) indicates the capture effect arising from near/far effect. Obviously, the capture effect is dominated by the Rayleigh fading and near/far effect rather than by the power assignment scheme. Next, with $k = 4$ and $L = 2$, we further observe that the maximum throughput has been improved from $1/e$ to 0.59 without near/far effect while to 0.72 with near/far effect, which indicates a remarkable improvement relative to the conventional slotted ALOHA system.

The effect of the number of power levels on the system throughput and delay with $k = 2$ under random power assignment, Rayleigh fading and near/far effect are presented in Figure 7(a) and

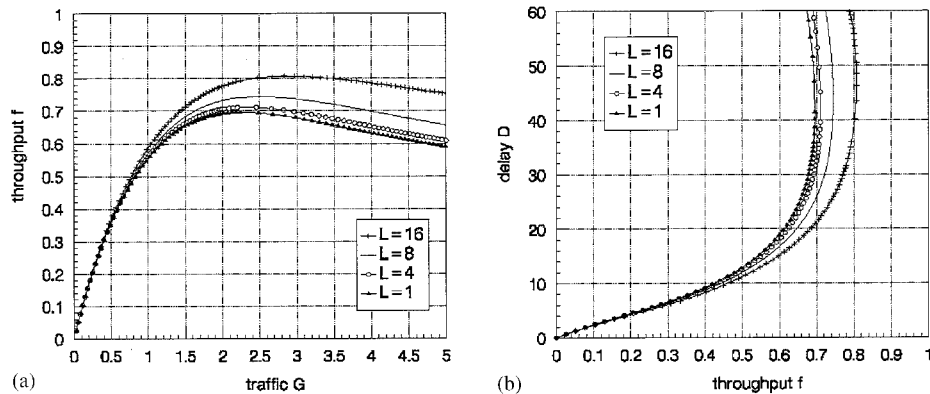


Figure 7. (a) f vs G under random power assignment, Rayleigh fading, and near/far effect with $k = 2$; (b) D vs f under random power assignment, Rayleigh fading, and near/far effect with $k = 2$.

(b), respectively. The phenomenon as L is increased is similar to that shown in Figure 6(a) and (b) with various values of k .

5. CONCLUSIONS

In this paper, we have studied the combined impact of random power assignment scheme, Rayleigh fading, and near/far effect on the performance of cellular mobile systems. With the assumption of finite population, we have successfully obtained the system performances by a Markov model. From the numerical results, it is found that system performance can be significantly improved by the combined natural and man-made capture effect. Moreover, the improvement amount due to random power assignment scheme is noticeable if the near/far effect disappears.

Herein, we only consider the power assignment scheme with random selection. The effect of other power assignment schemes, e.g. fixed power assignment, and adaptive power assignment, on cellular mobile system is worthy to further study. Moreover, although, in this paper, the power assignment technique is considered for single cell with finite population only, the technique can also be realized in the packet transmission mechanism of connectionless data service for any wireless mobile communications. It can also be incorporated into the call setup procedure of connection-oriented services. A natural extension which requires further study, is networking the fixed base station together, thus allowing users to move from one cell to another, like present-time cellular mobile systems. In this environment, the influence of co-channel interference and adjacent-channel interference should be taken into consideration and is deserving of further study. Therefore, our research is aimed to the extension of the analysis to the presence of both interferences.

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AUTHORS' BIOGRAPHIES



Jyh-Horng Wen was born in Taiwan, the Republic of China, on 12 May, 1957. He received the BS degree from the National Chiao Tung University, Hsing-Chu, Taiwan, in Electronic Engineering in 1979, and the PhD degree from the National Taiwan University, Taipei, Taiwan, in Electrical Engineering in 1990. From 1981 to 1983, he was a Research Assistant at the Telecommunication Laboratory, Ministry of Transportation and Communications, Chung-Li, Taiwan. From July 1983 to February 1991, he was a Research Assistant at the Institute of Nuclear Energy Research, Taoyun, Taiwan. Since February 1991 he has been an Associate Professor at the National Chung Cheng University, Chia-Yi, Taiwan. His current research interests include computer communication networks, cellular mobile and personal communications, interference cancellation technique, power control, access techniques, QoS of wireless broadband systems, and grey system.



Kuo-Gen Hsu received the MS degree in electrical engineering from National Chung-Cheng University, Chia-Yi, Taiwan in 1993. From 1988 to 1996, he worked as a senior Technical Staff at Data Communication Institute of Directorate General of Telecommunications (DGT), where he was work for the issues related to planning and OA&M of the data network such as X.25, Frame Relay, ATM network, and securities private network. Since the restructuring of the DGT in July 1, 1996 with the segregation of telecom supervision and operation, he transferred to the new DGT and served as a specialist. His areas of interest include wireless communication, broadband network and telecom policy. His current job is mainly on planning on the promotion of telecom liberalization and the opening to the telecom service markets. The main service items to be opened include: fixed networks, leased circuit and resale services. He undertook to draft the regulations governing the fixed network services, and these regulations had been proclaimed in May 18, 1999.



Jet-Chan Wen was born in Taiwan, Republic of China, 15 January 1959. He received the BE and the ME degree in Hydraulic Engineering from Chung-Yuan University, in 1980 and 1985, respectively. He finished the PhD degree in Civil & Environmental Engineering from Utah State University in 1993. From 1993 to 1997, he served as a faculty of the Department of Civil Engineering, Chung-Yuan University. Currently, he is an Associate Professor of the Department of Environmental & Safety Engineering, National Yunlin University of Science & Technology. His research interests include wireless communications, groundwater hydraulics, groundwater remediation, stochastic approaches on groundwater problems, and grey system on soil and water management. Dr Wen is a member of the American Society of Agronomy (ASA), the Soil Science Society of America (SSSA), the China Institute of Engineers (CIE), and the Chinese Institute of Environmental Engineering (CIEE).



Yi-Show Chen was born in Taiwan on 19 June 1973. She received a BS degree in electrical engineering from National Chung Cheng University, Chia-Yi in 1996 and a MS degree in electrical engineering from the same University in 1998. She is currently working in the Computer & Communications Research Laboratories of Industrial Technology Research Institute, Taiwan, as an associate engineer, where she is doing researches in 3G wide-band CDMA communication systems. Her current research interests include channel estimation, digital filters, and interference cancellation for wireless communication systems.