Estimation of effective aquifer hydraulic properties from an aquifer test with multi-well observations (Taiwan)

Jet-Chau Wen · Cheng-Mau Wu · Tian-Chyi J. Yeh · Chung-Ming Tseng

Abstract The representative anisotropic parameters for an aquifer located at the campus of the National Yunlin University of Science and Technology in Taiwan have been acquired. A constant-rate pumping test was carried out, and drawdown-time data were collected from ten observation wells. Applications of the conventional aquifer test analysis, which assumes aquifer homogeneity for each observed well hydrograph, yielded spatially varying transmissivity and storage coefficient estimates, contradicting the homogeneous assumption. A direct approach and a nonlinear-least squares minimization of distance-drawdown data were then employed to analyze anisotropy of the transmissivity of an equivalent homogeneous aquifer. Results show that the direction and values of anisotropic transmissivities vary with time and the estimates depend on the number of observation wells used. These field results are consistent with results from recent theoretical investigations which questioned the suitability of the conventional aquifer test analysis.

Keywords Heterogeneity · Anisotropic transmissivity · Multi-well system · Groundwater hydraulics · Taiwan

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Introduction

Several methods are available for determining horizontal aquifer anisotropy using drawdown time and several of these have specific requirements for the minimum number of observation wells and their alignment relative to the pumping well (Batu 1998). The Hantush and Thomas (1966) method, for example, requires observation wells to be placed along at least three different radial directions from the pumping well to determine the aquifer storage coefficient and the principal orientation and magnitude of the areal anisotropy of transmissivity. The Papadopoulos (1965) method, used for determining the transmissivity tensor of a homogeneous and anisotropic aquifer, requires three or more observation wells along different directions from the pumping well. This method is based on the assumption of an infinite areal extent of the aquifer and a constant-rate aquifer pumping test. Way and McKee (1982) proposed a method which could be used to investigate three-dimensional permeability in homogeneous, anisotropic, leaky formations. However, they did not present a procedure for determining three-dimensional permeability from field aquifer-test data. Neuman et al. (1984) demonstrated that the Papadopoulos method could be used with just three wells if separate aquifer tests were conducted in one of the wells, while in each case drawdowns were measured in the other two un-pumped (observation) wells. Kern and Dobson (1998) extended the method developed by Neuman et al. (1984) and proposed two statistical tests of anisotropy and variance formulas for construction of confidence intervals for the angles of orientation and the magnitudes of the principal transmissivities. For determining anisotropic

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hydraulic conductivity in aquifers, field determination of the three-dimensional hydraulic conductivity tensor of anisotropic media has been done by Hsieh and Neuman (1985) and Hsieh et al. (1985).

Radial flow in heterogeneous aquifers has been studied by many researchers in the past (see Meier et al. 1998). In particular, Butler and Liu (1993) derived an analytical solution for the case of transient, pumping-induced drawdown in a uniform aquifer into which a disk of anomalous properties—having a different transmissivity (T) and storativity (S)—has been placed. They found that changes in drawdown are sensitive to the hydraulic properties of a discrete portion of an aquifer for a time of limited duration. After that time, it is virtually impossible to gain further information about those properties. What was shown by Butler and Liu (1993) for a discrete feature was also shown by Oliver (1993) and Leven and Dietrich (2006) in a more general work—the former with a more theoretical work; the latter with more practical implications. There are others who have determined that the estimated flow parameters vary considerably from one observation point to the next. Illman and Neuman (2001, 2003) conducted a cross-hole pneumatic test in an unsaturated fractured tuff and used a type-curve interpretation for the data. Illman and Tartakovsky (2006) used an asymptotic analysis of cross-hole hydraulic tests in fractured granite. Recently, Mutch (2005) used scalar transformation to convert an anisotropic medium to an equivalent, isotropic medium, thus permitting application of the Cooper and Jacob (1946) method. He presented applicability of the method to both cases where at least one ellipse of equal drawdown could be delineated and where no ellipses could be discerned from the data. Heilweil and Hsieh (2006) used a simplified Papadopoulos method to determine anisotropic transmissivity. By assuming that observation wells are drilled along the principal directions from the pumping well, they reduced the requirements of observation wells from three to two.

Using a numerical simulation of a pumping test in a synthetic, two-dimensional, heterogeneous aquifer with a very large number of observation wells, Wu et al. (2005) used a spatial moment method to analyze the transmissivity anisotropy of an equivalent homogeneous aquifer. They reported that the magnitude and orientation of the anisotropy evolves with time but stabilizes after long periods of time when the cone of depression is large enough to cover sufficient heterogeneity. They subsequently challenged the meaningfulness of the anisotropy estimates using the aforementioned conventional anisotropy analysis based on just a few well hydrographs. In addition, they questioned the validity of the application of the Theis solution, which employs an equivalent homogeneous aquifer assumption to a single observation well in a heterogeneous aquifer. However, few field experiments have been conducted to corroborate their findings.

This paper aims to acquire the representative anisotropic, effective hydraulic properties of an aquifer in Taiwan, based on a field aquifer test which consists of a pumping well and ten observation wells. Pumping tests were carried out with a constant flux rate and drawdown-time data were collected from the ten observation wells. An optimization approach using drawdown data from ten observation wells simultaneously was applied to obtain a theoretically consistent anisotropic transmissivity, and the effects of the number of observation wells during an aquifer test on the estimates of the equivalent homogeneous hydraulic properties of a heterogeneous aquifer was illustrated.

Field site description

The aquifer test with multiple observation wells was conducted at an experimental field site on the campus of the National Yunlin University of Science and Technology (NYUST), located in the western central part of Taiwan. There were eleven wells at the site and each observation well was drilled 20 m deep from the ground surface. A Schedule 40 PVC pipe of 4.00-inch (10.2-cm) diameter was used for the well screen and casing, and the borehole was screened with a 0.02-inch (0.1-cm) slotted screen. The screen length was 18.50 m and was placed 1.50 m below the ground surface and extended down to the 20.00 m depth. A layout of the pumping and observation wells is shown in Fig. 1, with the pumping well denoted by BH04. Table 1 lists the coordinates of the pumping well and observation wells. The radial distance of each observation well from the pumping well is also presented in Table 1. During the pumping test, a constant flow rate of 1.78×10^{-4} cubic meter per second (m^3/s) was maintained for 4 days at the pumping well. The remaining ten wells were used as observation wells to monitor the drawdowns.

Data taken from the logs of the drilled wells and the grain-size analysis of five wells show that the aquifer is stratified with several geological layers: the depth 0.00–1.10 m is refilled soil of clay, sand, gravel and rocks; 1.10–3.40 m is silt-clay; 3.40–6.35 m is fine sand with fine gravel; 6.35–7.40 m is silt-clay with organics; 7.40–10.70 m is fine sand with clay; 10.70–14.60 m is silt-clay; 14.60–20.00 m is silt-sand with clay; and clay when the depth is greater than 20.00 m. Figures 2 and 3 show

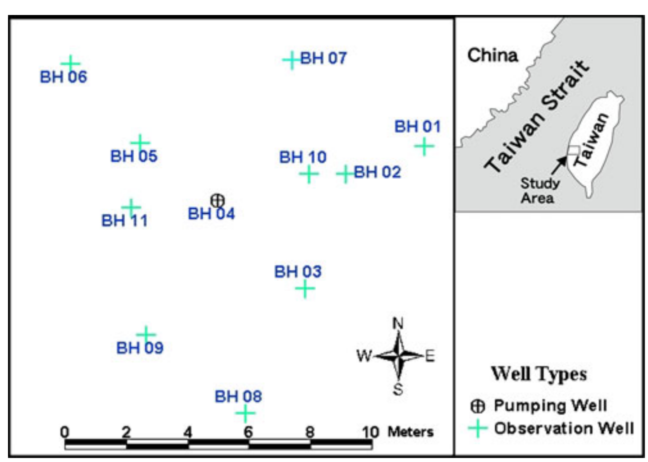

Fig. 1 Location map of the NYUST campus site and position of wells

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Table 1 Details of pumping well and observation wells

Well No.	x-coordinate	y-coordinate	Ground elevation (m asl)	Water level elevation in the well (m)	Distance x (m)	Distance y (m)	Radial distance from pumping well (m)
BH01	202785.76	2621451.04	49.39	45.56	6.68	1.83	6.92
BH02	202783.21	2621450.13	49.44	45.05	4.13	0.92	4.23
BH03	202781.85	2621446.41	49.25	45.26	2.78	−2.80	3.94
BH04[a]	202779.08	2621449.21	49.25	45.45	0	0	0
BH05	202776.49	2621451.15	49.15	45.28	−2.59	1.94	3.24
BH06	202774.21	2621453.73	49.14	45.29	−4.88	4.52	6.65
BH07	202781.46	2621453.87	49.39	45.47	2.38	4.66	5.23
BH08	202779.91	2621442.30	49.24	45.53	0.83	−6.91	6.96
BH09	202776.68	2621444.86	49.37	45.68	−2.40	−4.35	4.97
BH10	202782.03	2621450.13	49.17	45.49	2.95	0.92	3.09
BH11	202776.20	2621449.04	49.67	45.18	−2.88	−0.17	2.88

[a] Pumping well

geological cross-sections A–A′ and B–B′, respectively (refer to Fig. 1 for well locations). As indicated in the figures, the aquifer is mainly comprised of four materials, namely gravel, loam, sand, and silt and clay. There are many parallel stratification layers; the groundwater moves along the sand layer (Figs. 2b and 3b).

Methods of analysis

Analytical solution

The well hydrographs (drawdown-time data) of the ten observation wells were analyzed by several methods that employ an analytical solution described in the following. The distribution of drawdown around a fully penetrating pumping well in a homogeneous, confined anisotropic aquifer at a constant pumping rate is assumed to be governed by

$$T_{xx}\frac{\partial^2 s}{\partial x^2}+2T_{xy}\frac{\partial^2 s}{\partial x\partial y}+T_{yy}\frac{\partial^2 s}{\partial y^2}=S\frac{\partial s}{\partial t}-Q\delta(x)\delta(y) \quad (1)$$

where s is drawdown; T_{xx}, T_{yy} and T_{xy} are the components of the transmissivities; S is storage coefficient; Q is discharge rate; x, y are coordinates of an arbitrary set of axes with the well at the origin.

A method for Eq. 1 was given by Papadopoulos (1965) which describes drawdown in observation wells in a confined, homogeneous and anisotropic aquifer extending infinitely in lateral directions. This method for determination of horizontal aquifer anisotropy requires four wells, one for water withdrawal and three for drawdown observations. Neuman et al. (1984) extended the method to situations where the total number of wells is as few as three based on a minimum of two pumping tests. After a rotation of the coordinate system to align the coordinate axes along the principal directions of the transmissivity, Eq. 1 takes the form

$$T_\alpha\frac{\partial^2 s}{\partial\alpha^2}+T_\beta\frac{\partial^2 s}{\partial\beta^2}=S\frac{\partial s}{\partial t}-Q\delta(\alpha)\delta(\beta) \quad (2)$$

The relations between the coordinates (α, β) based on the principal directions and the original coordinates (x, y) are

$$\begin{aligned}\alpha&=x\cos\theta+y\sin\theta\\ \beta&=-x\sin\theta+y\cos\theta\end{aligned} \quad (3)$$

where θ is an angle of coordinates (α, β) and (x, y). Converting Eq. 2 to an equivalent isotropic system and changing its coordinates to polar coordinates, Eq. 2 becomes

$$T_e\left[\frac{\partial^2 s}{\partial r_e^2}+\frac{1}{r_e}\frac{\partial s}{\partial r_e}\right]=S_e\frac{\partial s}{\partial t}-Q\delta(r,\theta) \quad (4)$$

where $T_e^2=T_{xx}T_{yy}-T_{xy}^2$, $r_e^2=T_{xx}y^2+T_{yy}x^2-2T_{xy}xy$, and $S_e=\frac{S}{\sqrt{\left(T_{xx}T_{yy}-T_{xy}^2\right)}}=\frac{S}{T_e}$

The drawdown in the observation wells can be expressed as

$$s(r_e,t)=\frac{Q}{4\pi T_e}W(u_e) \quad (5)$$

where $W(u_e)$ is the well function and $u_e=\frac{r_e^2S_e}{4T_et}$. Equation 5 in terms of T_e and S_e is identical to the Theis solution for radial flow in a homogeneous, isotropic aquifer with infinite lateral extent.

Incorporating Eq. 5 with the anisotropic transmissivities, T_{xx}, T_{yy}, and T_{xy}, leads to the solution

$$s(x,y,t)=\frac{Q}{4\pi\sqrt{\left(T_{xx}T_{yy}-T_{xy}^2\right)}}W(u_e) \quad (6)$$

in which

$$u_e=\frac{S}{4t}\frac{\left(T_{xx}y^2-2T_{xy}xy+T_{yy}x^2\right)}{T_{xx}T_{yy}-T_{xy}^2} \quad (7)$$

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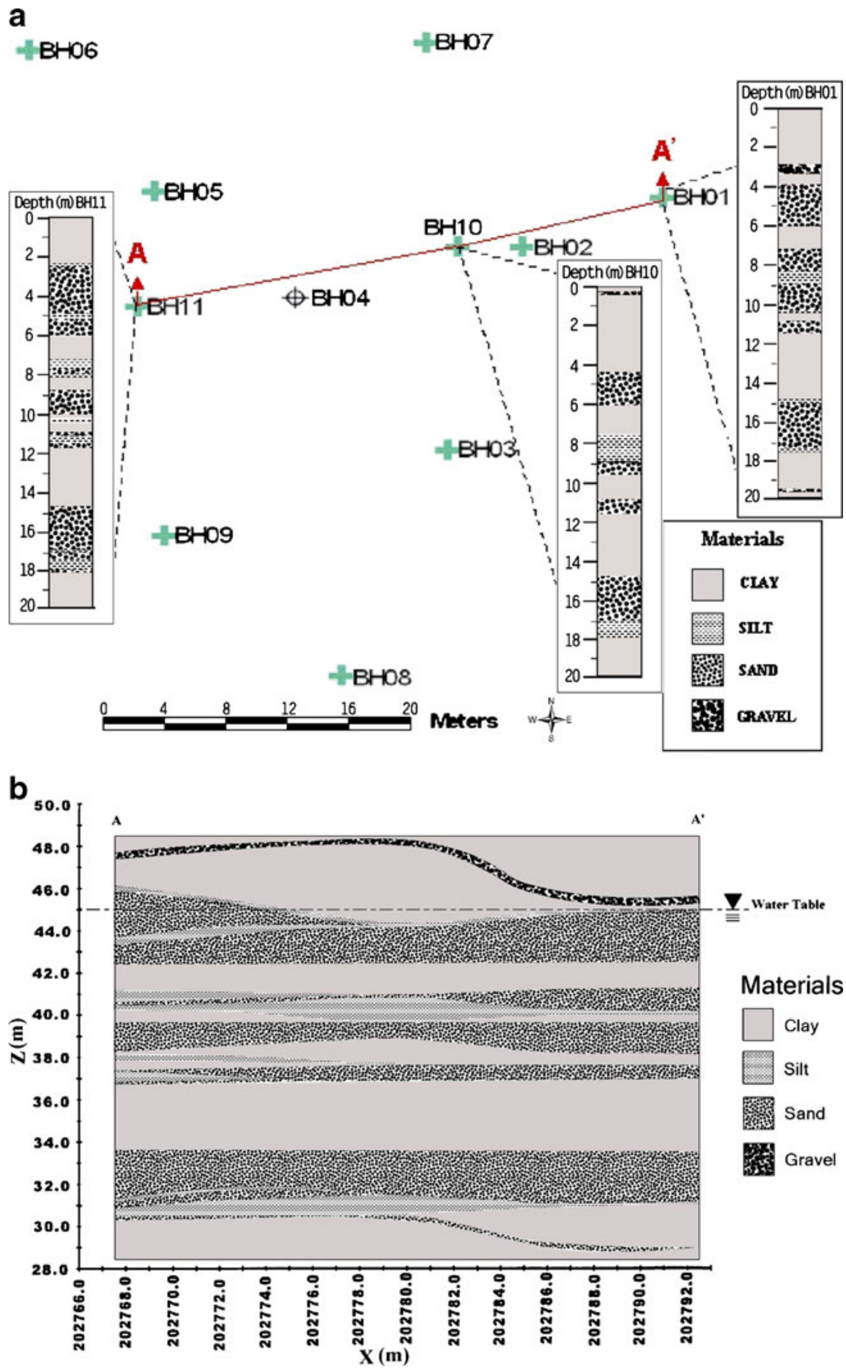

Fig. 2 **a** Stratigraphic cross sections across NYUST campus site (section *A–A′*). **b** The profiles of the aquifer along section A–A′

Equation 6 represents the theoretical drawdown at a location (x, y) in a confined, homogeneous and anisotropic aquifer.

Estimation of hydraulic properties

Using the aforementioned analytical solutions, the well hydrographs collected from the aquifer test were analyzed to estimate T and S for two scenarios: (1) when the aquifer is assumed to be homogeneous and isotropic and (2) when it is homogeneous and anisotropic.

Equivalent homogeneous S and isotropic T

To estimate T and S in the first scenario, a nonlinear least-squares approach was applied to minimize the following objective function:

$$\sum_i \left[\bar{s}(r, t_i) - s^*(r, t_i)\right]^2 = \text{minimal} \tag{8}$$

where $\bar{s}(r, t_i)$ and $s^*(r,t_i)$ are the theoretical drawdown in an equivalent homogeneous aquifer predicted by the Theis solution Eq. 5 and the observed drawdown at an observation

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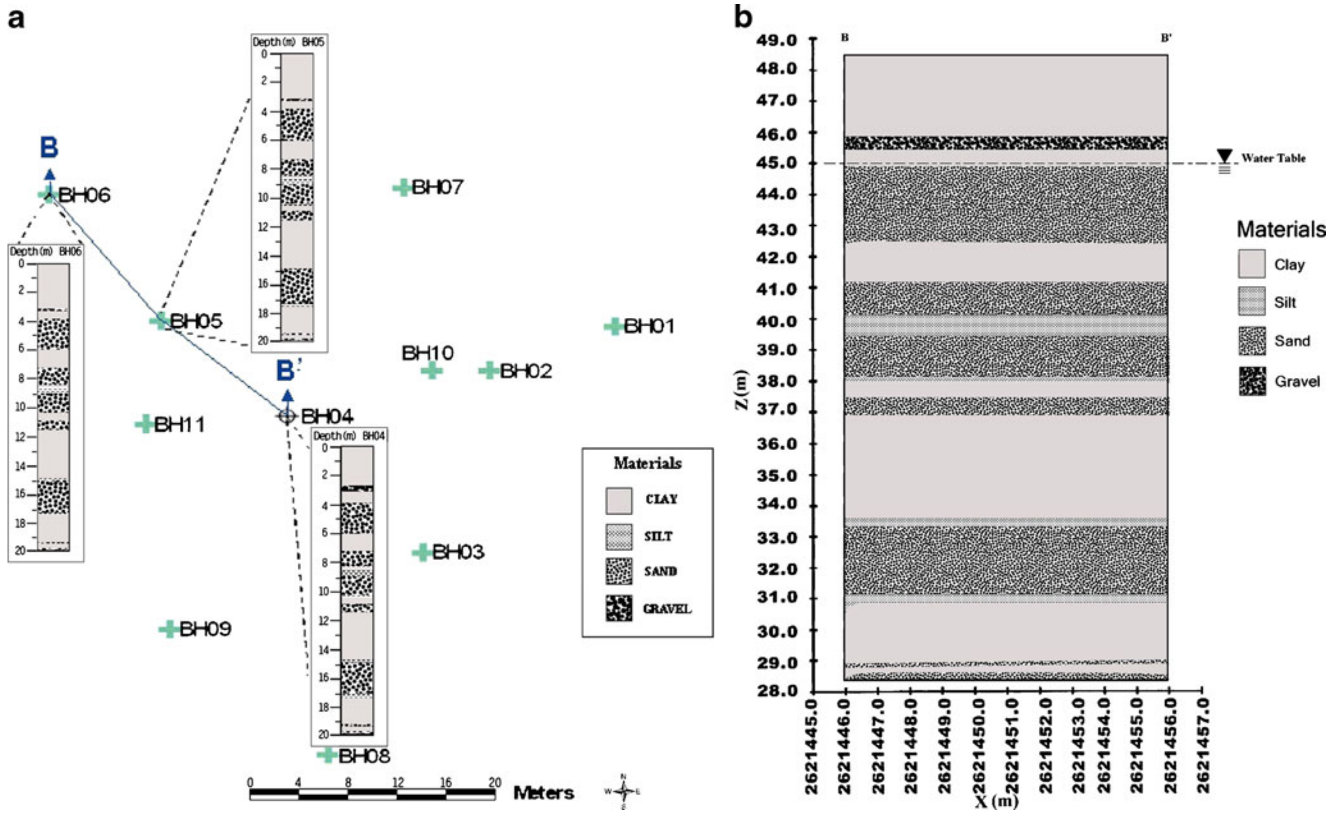

Fig. 3 **a** Stratigraphic cross sections across NYUST campus site (section *B–B′*). **b** The profiles of the aquifer along section B–B′

well at a distance r from the pumping well in an aquifer at a given time t_i, respectively. The objective function is calculated by including data at each sampling time interval, i, over a selected time period (e.g., early time, late time, or ensemble time (i.e., early and late times combined)) of the observed well hydrograph at a selected observation well.

Equivalent homogeneous S and anisotropic T

For scenario 2, two methods, namely direct and distance-drawdown, based on Eq. 6 were employed to determine the S and anisotropic T. They are discussed below.

Direct approach. One way to estimate the anisotropic transmissivity values, T_{xx}, T_{yy}, T_{xy}, and storage coefficient S is the approach by Papadopoulos (1965) and Neuman et al. (1984). It uses hydrographs from three observed wells to establish three equations with three unknowns, and then the system of equations is solved directly. In order to obtain a physically correct solution, the system of equations must meet a positive definite requirement.

Distance-drawdown approach. In this approach, a non-linear least-squares procedure is used to seek the optimal T_{xx}, T_{yy}, and T_{xy} and S which minimizes the following objective function:

$$\sum_i \left[\bar{s}(r_i, t) - s^*(r_i, t)\right]^2 \tag{9}$$

where $\bar{s}(r_i, t)$ and $s^*(r_i,t)$ are the theoretical drawdown predicted by Eq. 6 which assumes an equivalent homogeneous and anisotropic aquifer and the observed drawdown at an observation well at a distance r_i from the pumping well in an aquifer at a given time t, respectively. The objective function includes drawdowns at all observation wells at a given time. Wu et al. (2005) used this method to analyze well hydrographs generated in synthetic heterogeneous aquifers. They argued and demonstrated that this approach can yield theoretically consistent effective parameter values for equivalent homogeneous aquifers if a large number of observation wells are used.

In this study, the Gauss-Newton algorithm was employed to search the solution of the nonlinear least-squares minimization function Eq. 9. In order to apply the Gauss-Newton algorithm, the derivatives of Eq. 6 with respect to T_{xx}, T_{yy}, T_{xy} and S (i.e., sensitivity of drawdown to changes in parameters) were derived.

While McElwee and Yukler (1978) and Jiao and Rushton (1995) derived the derivatives for homogeneous isotropic aquifers, this paper presents the sensitivity for homogeneous, anisotropic aquifers. Using the Leibniz's integral rule and taking a partial derivative of s with respect to S and T, the sensitivity of drawdown s according to changes in S is

$$\begin{aligned}\frac{\partial s}{\partial S} &= \frac{Q}{4\pi\sqrt{\left(T_{xx}T_{yy} - T_{xy}^2\right)}}\frac{\partial W(u_e)}{\partial S}\\ &= \frac{-Q}{4\pi\sqrt{\left(T_{xx}T_{yy} - T_{xy}^2\right)}}\frac{e^{-u_e}}{S}\end{aligned} \tag{10}$$

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Similarly, the sensitivity of drawdown s according to changes in the anisotropic components, T_{xx}, T_{yy}, and T_{xy}, can be obtained by taking a partial derivative of s with respect to T_{xx}, T_{yy}, and T_{xy}, respectively, and they are

$$\begin{aligned}\frac{\partial s}{\partial T_{xx}} &= \frac{\partial}{\partial T_{xx}}\left[\frac{Q}{4\pi\sqrt{\left(T_{xx}T_{yy}-T_{xy}^2\right)}}W(u_e)\right] \\ &= \left(\frac{Q}{4\pi\sqrt{\left(T_{xx}T_{yy}-T_{xy}^2\right)}}\frac{\partial W(u_e)}{\partial T_{xx}}\right) \\ &\quad + \left(W(u_e)\frac{\partial}{\partial T_{xx}}\left(\frac{Q}{4\pi\sqrt{\left(T_{xx}T_{yy}-T_{xy}^2\right)}}\right)\right) \\ &= \frac{Q}{4\pi\sqrt{T_{xx}T_{yy}-T_{xy}^2}}\left(\frac{-y^2e^{-u_e}}{T_{xx}y^2-2T_{xy}xy+T_{yy}x^2}+\frac{T_{yy}(2e^{-u_e}-W(u_e))}{2\left(T_{xx}T_{yy}-T_{xy}^2\right)}\right)\end{aligned} \tag{11}$$

$$\begin{aligned}\frac{\partial s}{\partial T_{xy}} &= \frac{\partial}{\partial T_{xy}}\left[\frac{Q}{4\pi\sqrt{\left(T_{xx}T_{yy}-T_{xy}^2\right)}}W(u_e)\right] \\ &= \left(\frac{Q}{4\pi\sqrt{\left(T_{xx}T_{yy}-T_{xy}^2\right)}}\frac{\partial W(u_e)}{\partial T_{xy}}\right) \\ &\quad + \left(W(u_e)\frac{\partial}{\partial T_{xy}}\left(\frac{Q}{4\pi\sqrt{\left(T_{xx}T_{yy}-T_{xy}^2\right)}}\right)\right) \\ &= \frac{Q}{4\pi\sqrt{T_{xx}T_{yy}-T_{xy}^2}}\left(\frac{2xye^{-u_e}}{T_{xx}y^2-2T_{xy}xy+T_{yy}x^2}+\frac{T_{xy}(1-2e^{-u_e})}{2\left(T_{xx}T_{yy}-T_{xy}^2\right)}\right)\end{aligned} \tag{12}$$

and

$$\begin{aligned}\frac{\partial s}{\partial T_{yy}} &= \frac{\partial}{\partial T_{yy}}\left[\frac{Q}{4\pi\sqrt{\left(T_{xx}T_{yy}-T_{xy}^2\right)}}W(u_e)\right] \\ &= \left(\frac{Q}{4\pi\sqrt{\left(T_{xx}T_{yy}-T_{xy}^2\right)}}\frac{\partial W(u_e)}{\partial T_{yy}}\right) \\ &\quad + \left(W(u_e)\frac{\partial}{\partial T_{yy}}\left(\frac{Q}{4\pi\sqrt{\left(T_{xx}T_{yy}-T_{xy}^2\right)}}\right)\right) \\ &= \frac{Q}{4\pi\sqrt{T_{xx}T_{yy}-T_{xy}^2}}\left(\frac{-x^2e^{-u_e}}{T_{xx}y^2-2T_{xy}xy+T_{yy}x^2}+\frac{T_{xx}(2e^{-u_e}-W(u_e))}{2\left(T_{xx}T_{yy}-T_{xy}^2\right)}\right)\end{aligned} \tag{13}$$

Results

Analysis of well hydrographs of individual observation wells

Figure 4 illustrates the contours of measured drawdowns at the ten observation wells after four days of pumping for a constant flow-rate test in the aquifer. These contours show irregular drawdown distribution, indicating the effect of heterogeneity and the general behavior of an equivalent homogeneous aquifer with a transmissivity anisotropy (i.e., the drawdown distribution is elongated in the y direction, suggesting that T_{yy} is greater than T_{xx}).

The drawdown-time curve at individual observation wells shows that the observed drawdown time data agree with the theoretical Theis solution (solid lines) at early time (i.e. $t < 1 \times 10^4 sec$) and there exists a large discrepancy between the Theis solution and observed drawdown data for $t > 1 \times 10^4 sec$ (Fig. 5). At this late time period, most observed drawdowns are much less than those calculated by the Theis solution, indicative of possible effects of recharge boundary, leakage, thickness, or source of water from the unsaturated zone.

In spite of these deviations from the ideal behavior of Theis' aquifers at late time, the conventional analysis of an aquifer test, Eq. 5, assuming aquifer homogeneity and isotropy, was applied to the hydrograph of each observation well to estimate T and S (i.e., T_e and S_e). A comparison of the estimated parameters is presented in Table 2 using different observation times, i.e., (1) the early time of the drawdown data sets, (2) the late time of the drawdown data sets, and (3) the ensemble time of the drawdown data sets. Comparing with the estimates based on the early time data, transmissivity, on average, was overestimated by 131% when using the late-time observation data and by 17% when using the ensemble-time observation data. On the other hand, the estimated storage coefficient based on the late time data is 81% smaller on average and the estimate using the ensemble data is 10% smaller. For this study, the early time data were selected to estimate the parameters.

Table 2 also shows that the ten estimated T and S values vary with the observation well. Therefore, they are inconsistent with the homogeneous assumption embedded in the Theis solution. These spatially varying T and S values are plotted and contoured in Figs. 6 and 7, respectively. Wu et al. (2005) attributed this variability to the fact that application of the Theis analysis to the pumping test in heterogeneous aquifers yields a T estimate that represents an ambiguous average of heterogeneity within the cone of depression. This average is heavily influenced by local geology near the pumping well and each individual observation well. On the other hand, each S estimate is mainly controlled by the geology between the pumping well and the observation wells. The small value of the coefficient of variation in the transmissivity estimates (Table 2) and its smooth contours in Fig. 6 support their findings as does the fact that the contours of the transmissivity estimates appear to be smoother than the contours of the S estimates (Fig. 7). Table 2 also

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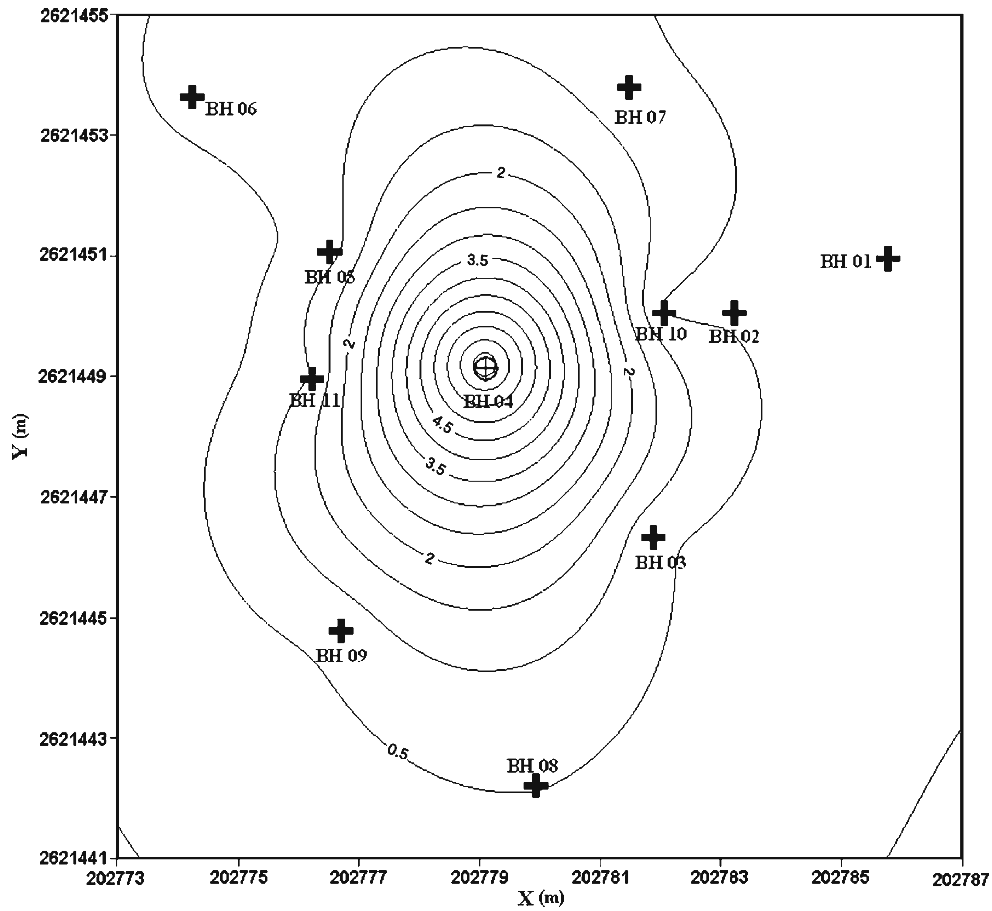

Fig. 4 Contours of drawdown (m below water surface) after 4 days of pumping

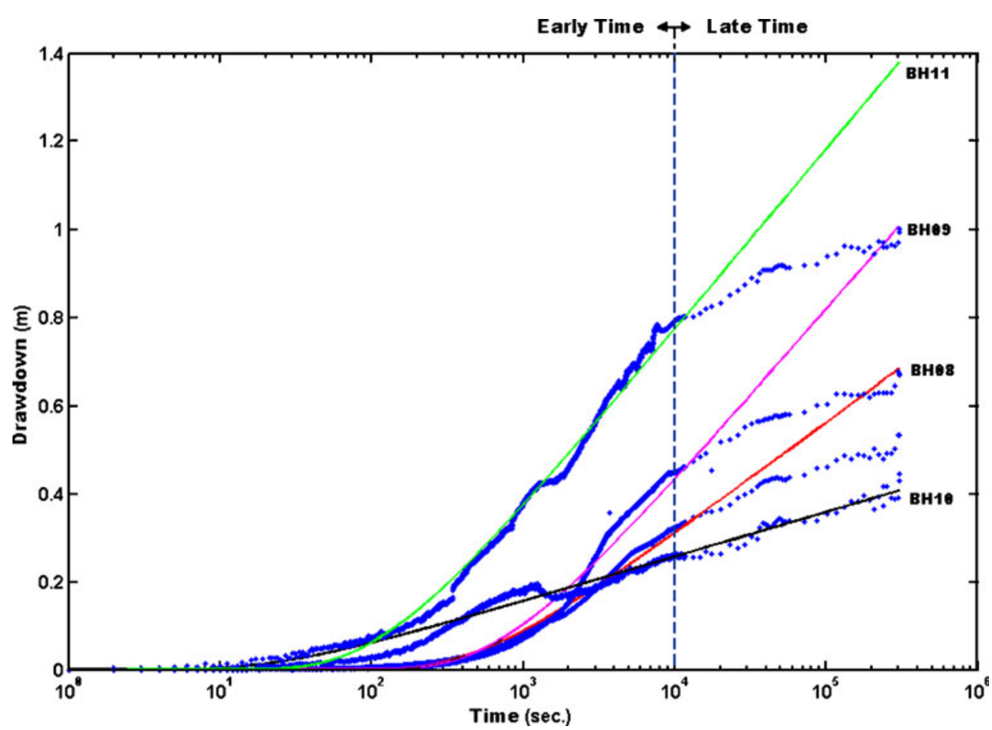

Fig. 5 The drawdown-time curve at individual observation wells (log scale). The *solid lines* represent theoretical Theis solution

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Table 2 Estimated parameter values from Theis' type-fitting curve

Well No.	Estimated using early time data		Estimated using late time data		Estimated using ensemble time data		Difference of estimation in percentage (%)			
	(1)[d] T^{e} (m^2/s)	(2) S	(3) T (m^2/s)	(4) S	(5) T (m^2/s)	(6) S	[(1)-(3)] /(1)×100	[(2)-(4)] /(2)×100	[(1)-(5)] /(1)×100	[(2)-(6)] /(2)×100
BH01	0.000097	0.007918	0.000213	0.001684	0.000125	0.007027	-119	79	-29	11
BH02	0.000142	0.009833	0.000295	0.000806	0.000159	0.008854	-108	92	-12	10
BH03	0.000086	0.009367	0.000241	0.000214	0.000103	0.008157	-180	98	-19	13
BH05	0.000072	0.013989	0.000195	0.000495	0.000087	0.012198	-172	96	-21	13
BH06	0.000086	0.004836	0.000194	0.000526	0.000103	0.004300	-126	89	-20	11
BH07	0.000153	0.002094	0.000205	0.000430	0.000152	0.002080	-34	79	0.07	0.7
BH08	0.000109	0.004097	0.000244	0.000405	0.000129	0.003621	-124	90	-18	12
BH09	0.000061	0.007351	0.000220	0.000173	0.000083	0.006239	-262	98	-36	15
BH10	0.000322	0.002161	0.000315	0.002491	0.000322	0.002158	2.15	-15	-0.07	0.1
BH11	0.000072	0.003322	0.000205	0.000006	0.000079	0.002822	-185	100	-11	15
Mean[a]	0.001199	0.006497	0.000233	0.000723	0.000140	0.005746	-131	81	-17	10
Var[b]	5.948E-09	1.5E-05	1.78E-09	6.01E-07	5.144E-09	1.12E-05				
CV[c]	0.06433	0.59621	0.18127	1.07216	0.51073	0.58146				

indicates that the coefficient of variation of S is about ten times that of T.

Estimation of anisotropy using the direct approach

Next, the direct approach was applied to estimate the anisotropic transmissivities T_{xx}, T_{yy}, and T_{xy}, respectively, using the early time hydrographs from different sets of combinations of three observation wells. Each set is composed of one pumping well with three observation wells. The different sets are characterized by the average radial distance between the observation wells and the pumping well. The average radial distance is the distance that averages over the three radial distances of the observation wells of each set. The estimated anisotropic transmissivity, T_{xx}, T_{yy}, and T_{xy}, varied randomly with the radial distance of the 120 combinations of observation wells. Some combinations of three wells led to negative T_{xx} and T_{yy} values, which are physically impossible, although negative values of T_{xy} are permissible because of coordinate transformation. These physically implausible results are mainly caused by (1) inconsistency between the theoretical drawdown of Eq. 6, which assumes aquifer homogeneity and the observed drawdown in heterogeneous aquifers (Wu et al. 2005), (2) presence of noise, and lastly, (3) inflexibility of the direct approach which requires intact head information. Therefore, these results are not presented in this paper.

Estimation of anisotropy using the distance-drawdown approach

To overcome the problem of the direct approach, the distance-drawdown minimization approach was applied to the hydrographs from the ten observation wells simultaneously at some specified times to estimate the anisotropic transmissivities and specific storage of the aquifer. This distance-drawdown approach using a large number of observation wells, as advocated by Wu et al. (2005), is consistent with the homogeneous aquifer assumption embedded in the Theis solution. In other words, the spatially averaged drawdown distribution based on a large number of wells is equivalent to the ensemble-average drawdown built in the Theis solution. In addition, the optimization approach avoids the difficulty of the direct method if drawdown data deviate significantly from the theoretical solution due to either heterogeneity or noise.

The estimated anisotropic T_{xx}, T_{yy}, T_{xy} values using different time spans of ten observed drawdowns are shown in Fig. 8 as a function of time. The estimated anisotropic parameters vary significantly at early time, and then they converge to constant values at some point after a

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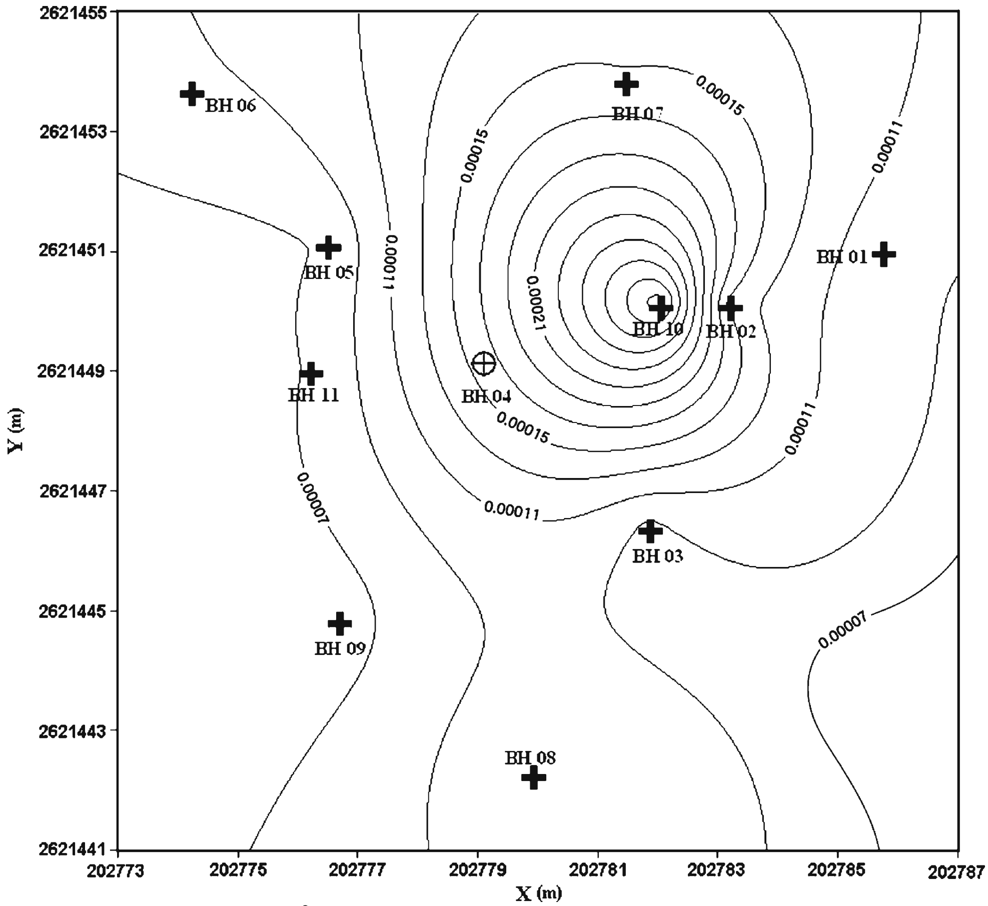

Fig. 6 Contours of estimated transmissivity (m^2/s) from individual observation wells

long pumping time. The values of T_{yy} are much larger than T_{xx} at early time, and then become only slightly larger than T_{xx} at late time. The estimated T_{xy} values also vary with time and are always negative, indicating that the principal directions of the effective transmissivity tensor change with time and are not in alignment with the x–y coordinate system chosen for this analysis. Figure 9 shows estimated S values at different time spans of the ten observed drawdowns. Again, the values fluctuate significantly at early time and stabilize at late time. Results of Figs. 8 and 9 perhaps suggest the existence of time-invariant effective hydraulic properties at this site for this pumping well after pumping for 297,900 sec before other effects (such as boundaries) took place.

To illustrate the effects of the number of observation wells during an aquifer test on the estimates of the equivalent homogeneous hydraulic properties of a heterogeneous aquifer, Fig. 10 shows the estimated anisotropic T_{xx}, T_{yy}, T_{xy} values using only four observed drawdowns. The four observation wells were a subset randomly selected from the ten observation wells. Contrary to the results of ten observed drawdowns (Fig. 8), the values of T_{yy} are smaller than T_{xx} at ensemble time. Figure 11 presents estimated S values at different time spans of the four observed hydrographs.

Comparisons of anisotropic transmissivity estimated from ten observation wells and four observation wells indicate that T_{xx} estimated from four observation wells represents a trend similar to the one estimated from ten observation wells. Both estimated T_{xx} values showed high fluctuations at early time. The estimates based on four wells are smaller than those based on ten wells but they tend to converge to a constant value at late time. The estimated T_{xy} values using four observation wells are smaller than the ones using ten observation wells. On the other hand, T_{yy} estimated using four observation wells are much smaller than the ones using ten observation wells. Results of these comparisons manifest the fact that in order to obtain representative, equivalent homogeneous hydraulic properties of heterogeneous aquifers, a sufficient number of observation wells and a sufficiently long aquifer test should be undertaken. A distance-drawdown analysis would be the most appropriate method as suggested by Wu et al. (2005).

Finally, many have claimed that application of the conventional analysis, which assumes aquifer homogeneity, to aquifer tests with one pumping well and one observation well yields a spatially averaged transmissivity, although cautious interpretation has been emphasized (e.g., Butler

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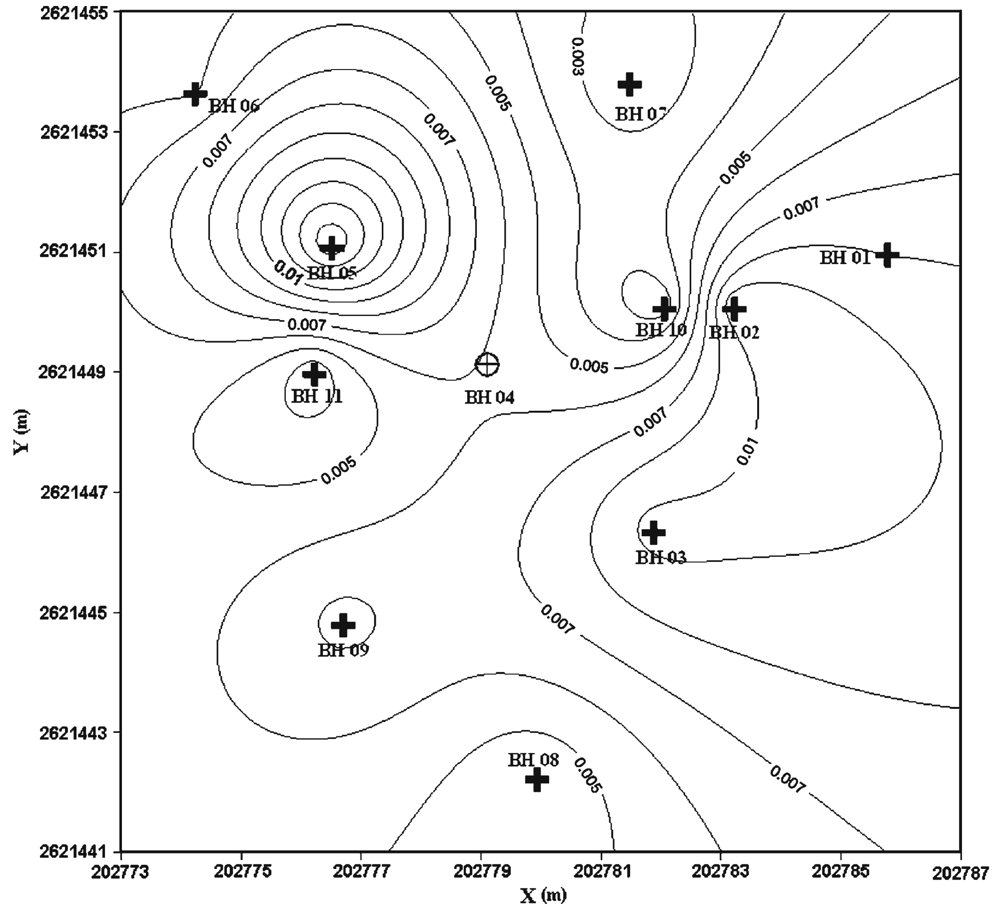

Fig. 7 Contours of estimated storage coefficient from individual observation wells

1990; Oliver 1993). The results of this study and those by Wu et al. (2005) and Straface et al. (2007) nonetheless demonstrate that the analysis results in an evolving transmissivity estimate, which is time dependent as the cone of depression grows. While the estimate represents some type of average of spatially varying local transmissivity within the cone of depression, it is influenced by transmissivities near the pumping and observation wells. On the other hand, the

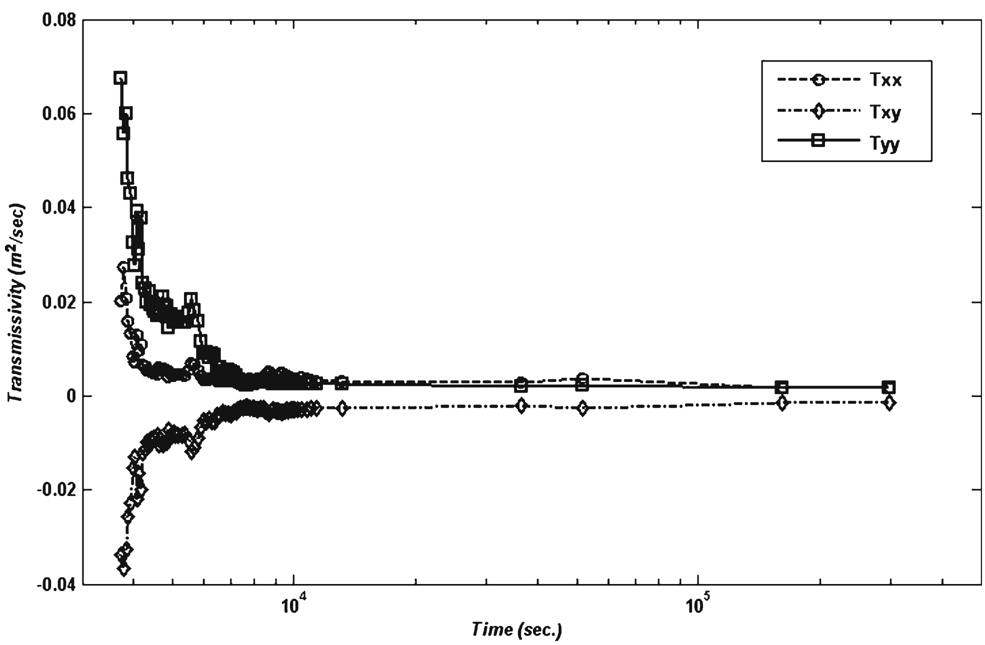

Fig. 8 Variations of estimated T_{xx}, T_{yy}, T_{xy} vs. time (using ten observation wells)

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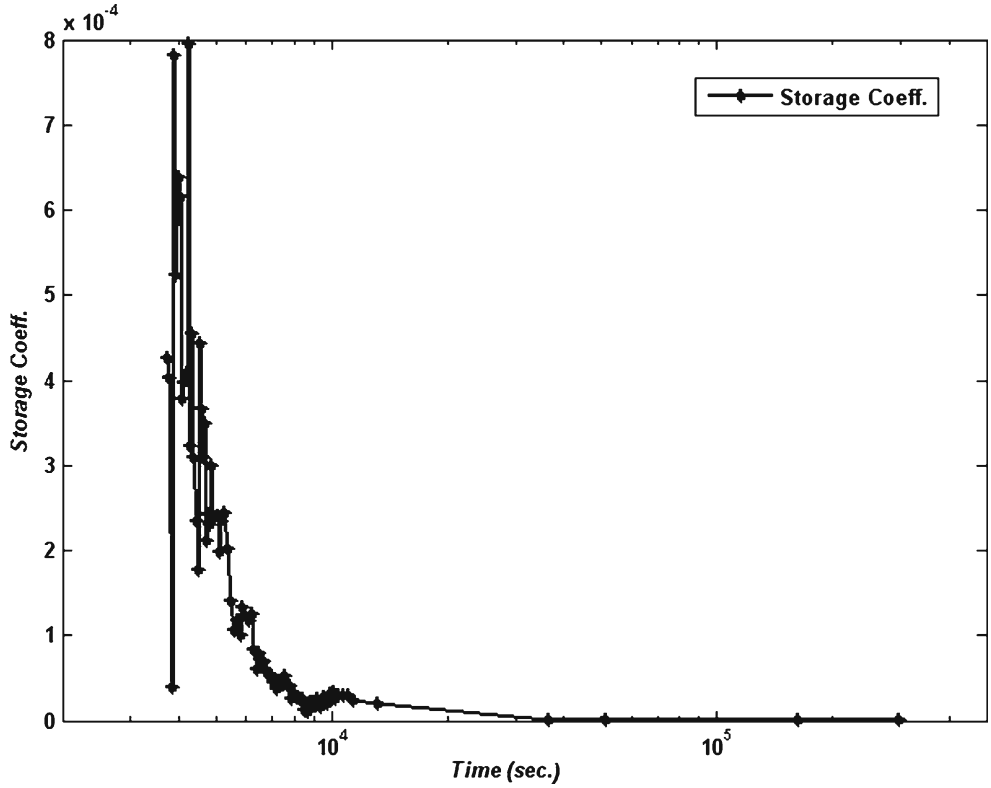

Fig. 9 Variations of estimated S vs. time (using ten observation wells)

storage coefficient reflects some averaged storage properties between the pumping well and the observation wells. Similar results have been reported by Schad and Teutsch (1994), Meier et al. (1998), Leven and Dietrich (2006) and by Illman et al. (2007) and Liu et al. (2007) using well-controlled sand box experiments.

Likewise, it has been taught that an aquifer is leaky or unconfined (e.g., Hantush 1956) based on the resemblance

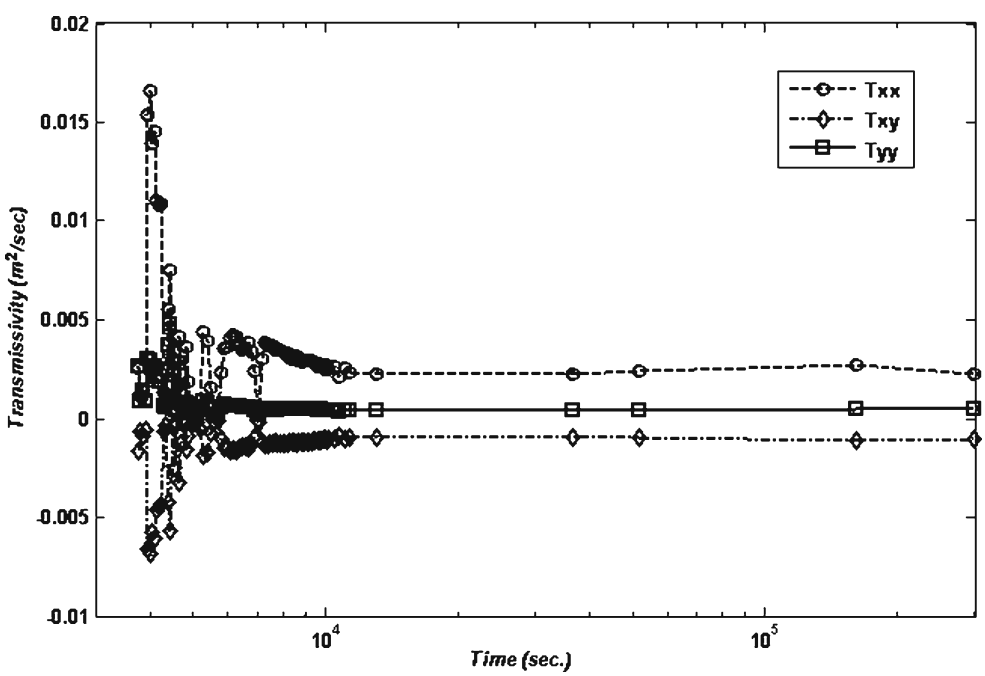

Fig. 10 Variations of estimated T_{xx}, T_{yy}, T_{xy} vs. time (using four observation wells)

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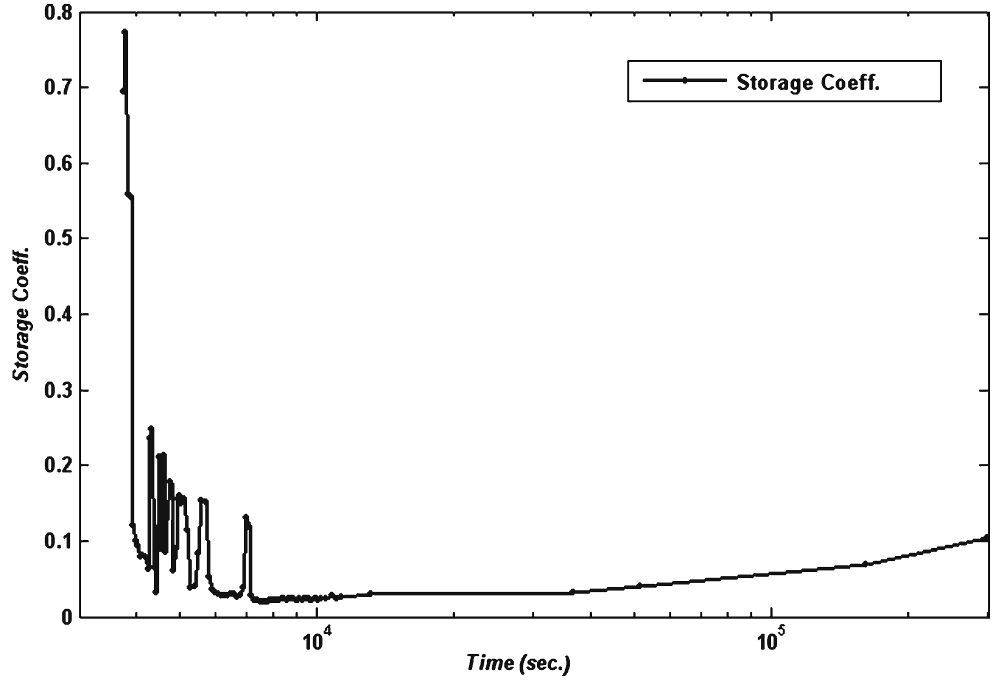

Fig. 11 Variations of estimated S vs. time (using four observation wells)

between one single hydrograph and the solution of the leaky aquifer equation or delayed drainage model (e.g., Boulton 1954; Neuman 1972; Moench 1997) that assumes homogeneity. In fact, aquifer heterogeneity and many other factors may yield similar behaviors. These inconsistent, although practical, applications of simplified theories to complex real-world geologic media may have profoundly impacted groundwater resource investigations.

Summary and conclusions

The analysis of a pumping test at a ten-observation well site on the campus of NYUST showed that application of the Theis solution to the aquifer test that is based on a well hydrograph from an observation well does not yield unique averaged T and S values. In other words, based on an individual well-hydrograph, the equivalent isotropic, homogeneous parameters are not possible to obtain. This result confirms the finding by Wu et al. (2005) that the traditional analysis results in an evolving transmissivity estimate, which is time dependent as the cone of depression grows. While the estimate represents some type of average of spatially varying local transmissivity within the cone of depression, it is influenced by transmissivities near the pumping and observation wells. On the other hand, the storage coefficient reflects some averaged storage properties between the pumping well and the observation wells.

Additionally found was that the traditional three-observation well approach for estimating transmissivity anisotropy yielded anisotropic transmissivity values that vary randomly with the radial distance when 120 combinations were made of the observation wells. A large number of estimated transmissivity values were negative, indicative of physically incorrect values. This is attributed to inconsistency between theoretical and observed drawdowns due to noise and heterogeneity as well as inflexibility of the direct approach.

Alternative to the direct approach, an optimization approach using drawdown data from ten observation wells simultaneously was applied to obtain a theoretically consistent anisotropic transmissivity. The results show that the anisotropy also evolves with time and the extent of the cone of depression. In addition, the estimates of T_{xx} and T_{yy} from a subset of the ten observed hydrographs can be entirely different from those derived from the entire set of ten hydrographs. These results support the conjecture by Wu et al. (2005) that, since aquifers are inherently heterogeneous, applications of the Theis method or other methods that assume aquifer homogeneity require a sufficient number of observation wells as well as hydrographs of long records in order to obtain representative homogeneous aquifer properties. These results also echo the call by Yeh and Lee (2007) for changing the way data are collected and analyzed for characterizing aquifers.

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