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Effects of Raingauge Distribution on Estimation Accuracy of Areal Rainfall

Chun-dan Cheng · Shin-jen Cheng · Jet-chau Wen · Ju-huang Lee

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Abstract Rainfall analysis is important to managing water resources. Mean rainfall is usually used to calculate the spatial rainfall status of a region and is the input into various rainfall-runoff models. However, this method relies on an adequate raingauge network. This study identifies the effects of raingauge distribution based on estimation results of areal rainfall using the Thiessen polygon and block Kriging methods. Twelve rainfall events with complete data from 14 raingauges were selected to complete the goal of this study. The block Kriging method in this study uses a dimensionless semivariogram to obtain hourly semivariograms based on a standardized rainfall depth. The power semivariogram model was used to describe the temporal-spatial variation of rainfall. The analytical process in this study uses raingauge weight and rainfall volume as evaluation criteria. All raingauges were in turn removed from the original raingauge network. The effects of removing each raingauge were compared with computations using all raingauges. Comparison results indicate that (1) the block Kriging method can accurately describe rainfall processes in terms of the spatiotemporal structure of a semivariogram. (2) the block Kriging method is better than the Thiessen polygon method at obtaining exact mean rainfall, and (3) the effects of different raingauge distributions on a mean hyetograph warrant further investigation.

C.-d. Cheng
Graduate School of Engineering Science and Technology, National Yunlin University of Science and Technology, Douliou, Yunlin 640, Taiwan

S.-j. Cheng (✉)
Department of Leisure and Recreation Management, Taiwan Shoufu University, 168, Nan-shih LiMadou 721 Tainan, Taiwan
e-mail: sjcheng@tsu.edu.tw

J.-c. Wen
Department of Safety Health and Environmental Engineering, National Yunlin University of Science and Technology, Douliou, Yunlin 640, Taiwan

J.-h. Lee
Water Resources Agency, Ministry of Economic Affairs, Taipei 106, Taiwan

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1 Introduction

Heavy rainfall events frequently hit Taiwan during the typhoon season, and typically generate considerable river runoff, countless overland flows, landslides, debris flows, and power outages. The magnitude of rainfall events in Taiwan has increased recently, likely due to climate change. This has increased the need for a faultless system for flood modeling. However, some uncertainties, such as missing rainfall data, can adversely affect rainfall and runoff estimations during typhoons. These uncertainties should be considered in any flood modeling system to help manage emergencies, accurately estimate flood hydrographs, and mitigate loss of life and property during heavy rainfall events.

Recording meteorological data is a common task in managing regional water resources. Hydrological analysis, and especially that for flood modeling, uses mean rainfall data. In conceptual hydrological models, many hydrologists have derived various instantaneous unit hydrograph (IUH) forms (Agirre et al. 2005; Ahmad et al. 2009; Cheng 2010a) to analyze watersheds in terms of linear cascade reservoirs in parallel or serial. These models conceptually describe the catchment responses of excess rainfall.

Mean rainfall of a watershed or its divisions is usually entered into conceptual models for routing runoff hydrographs. There are several schemes for estimating average rainfall values from raingauge data, including the inverse distance method (Maidment 1993), Thiessen polygon method, height-balanced polygon method, and block Kriging method (Delhomme 1978; Isaaks and Srivastava 1989; Cheng and Wang 2002; Cheng et al. 2008b; Huang et al. 2008a; Huang et al. 2008b; Cheng et al. 2010; Cheng 2010b, c). These methods often assign weights to raingauge observations using weighted mean by related observations, and then obtain mean rainfall estimations for specific areas. All of these methods consider that the sum of weights equals one for an unbiased estimation. The block Kriging method has the further advantage of being able to minimize the estimated error variance (Wackernagel 1998; Chiles and Delfiner 1999).

Hydrologists frequently have no choice but to evaluate areal rainfall roughly when applying other hydrological aspects due to the need for hydrological data for sites without raingauges or data missing. Because these weights are typically calculated according to raingauge locations or spatial variation (a semivariogram in the Kriging method), weights are fixed for each event. However, missing data makes it necessary to recalculate the weights of raingauges by removing inoperable gauges to minimize estimation bias. These redistributed weights differ from the original raingauge network. Thus, an insufficient number of raingauges in a network can adversely affect the accuracy of rainfall-runoff analysis.

Theoretically, as the number of raingauges increases, the accuracy of rainfall estimation increases. However, only a limited number of raingauges can be set up due to cost considerations and geographical limitations. An adequate raingauge network contributes significantly to the spatiotemporal analysis of rainfall (Berne et al. 2004). Rainfall spatial variability is also a concern in runoff modeling (Faurès et al. 1995; Arnaud et al. 2002; Syed et al. 2003). In the past, most hydrologists have specialized in designing raingauge networks (Bastin et al. 1984; Lebel et al. 1987; Cheng et al. 2008a). This study focuses on assessing the number of representative raingauges required by various rainfall estimation methods. The analytical process adopted in this study removes each raingauge in turn from an existing raingauge network, and then compares the utility of the Thiessen polygon and block Kriging methods. Finally, variations in the mean hyetographs resulting from missing rainfall data were used to identify the significant raingauges in an existing raingauge network.

2 Methods

Rainstorms vary markedly in space and time. The spatially representative resulting from rainfall variation is represented by important raingauges, and relative weights may then be assigned to these raingauges for calculating areal rainfall. Areal rainfall derived using these representative sites is typically represents the rainfall characteristics of that region. Traditional methods, such as the Thiessen polygon method, are frequently used to compute mean rainfall. For the block Kriging method, a semivariogram with a spatial relationship can be used to describe rainfall variation in space and to determine the point or areal rainfall via the block Kriging system (Wackernagel 1998; Chiles and Delfiner 1999; Cheng et al. 2007).

2.1 Thiessen Polygon Method

The Thiessen method assumes that at any point in the watershed the rainfall is the same as that at the nearest gauges. Thus, the depth recorded at any given gauge is carried out to a distance halfway to the next station in any direction. The relative weights for each gauge are determined from the corresponding areas of application in a Thiessen polygon network. The boundaries of the polygons are formed by the perpendicular bisectors of the lines joining adjacent gauges (Chow et al. 1988; Maidment 1993).

2.2 Block Kriging Method

The set of time sequences of discontinuous point-rainfall depths $p(t,x)$ can be viewed as a realization of two-dimensional random fields. Considering n raingauges in a river basin, for each time period t, a realization $\pi(t)$ of the random n vector can be expressed as

$$\pi(t) = \{p(t,x_1), p(t,x_2), \ldots, p(t,x_n)\} \tag{1}$$

where $p(t,x_n)$ is the rainfall measured by the n-th raingauge at the t-th time period. For rainfall depth $p(t,x)$, the intrinsic hypothesis is

$$m(t,x) = \mathrm{E}[p(t,x)] \tag{2}$$

$$\gamma(t,h_{ij}) = \gamma(t,x_i,x_j) = \frac{1}{2}\mathrm{E}\left\{\left[p(t,x_i) - p(t,x_j)\right]^2\right\} \tag{3}$$

where $\gamma(t,h_{ij})$ denotes a semivariogram of raingauges x_i and x_j, and h_{ij} is distance between arbitrary raingauges x_i and x_j. The experimental semivariogram of rainfall depth can be computed using Eq. 4

$$\gamma(t,h_{ij}) = \frac{1}{2T}\sum_{t=1}^{T}\left\{\left[p(t,x_i) - p(t,x_j)\right]^2\right\} \tag{4}$$

where T is the total duration of all rainfall events (hours). Bastin et al. proposed a basic semivariogram, called the scaled climatological mean semivariogram, in 1984. Cheng et al. (2007) later established this semivariogram using dimensionless rainfall data for a given basin. That basic experimental semivariogram is

$$\gamma(t, h_{ij}) = \omega(t)\gamma_d^*(h_{ij}, a) = s^2(t)\gamma_d^*(h_{ij}, a) \tag{5}$$

in which

$$\gamma_d^*(h_{ij}, a) = \frac{1}{2T}\sum_{t=1}^{T}\left\{\left[\frac{p(t, x_i) - p(t, x_j)}{s(t)}\right]^2\right\} \tag{6}$$

where $\omega(t)$ is the semivariogram sill for time period t (mm^2), $\gamma_d^*(h_{ij}, a)$ denotes the scaled climatological mean semivariogram (mm^2) and is time-invariant, a is the range of the scaled climatological mean semivariogram (m), $p(t,x_i)$ is rainfall at raingauge x_i for time period t (mm), and $s(t)$ is the standard deviation of rainfall at all raingauges for time period t (mm).

This basic experimental semivariogram can be estimated using Eq. 6. Because an experimental semivariogram is based on discontinuous point observations, it is not spatially continuous. This study uses a realistic application of the block Kriging method with a semivariogram model (the power model) to obtain the spatial continuity of rainfall variations in Taiwan. The expression of the power model is as follows:

$$\gamma(h) = \omega_0 h^a, a < 2 \tag{7}$$

where ω_0 is the sill of the scaled climatological mean semivariogram (mm^2) and is a constant of approximately one except for the power model.

Optimal weights were obtained using the Kriging system by assuming a given spatial rainfall structure. The system was obtained by applying following Lagrange multipliers as follows:

$$\begin{cases} \sum_{j=1}^{n}\lambda_j\gamma(x_i, x_j) + \mu = \overline{\gamma}(V, x_i), & i = 1, 2, \ldots, n \\ \sum_{i=1}^{n}\lambda_i = 1 \end{cases} \tag{8}$$

$$\sigma_K^2 = \sum_{i=1}^{n}\lambda_i\overline{\gamma}(V, x_i) + \mu \tag{9}$$

where $\gamma(x_i,x_j)$ is the semivariogram of raingauges x_i and x_j (mm^2), $\overline{\gamma}(V, x_i)$ represents the average semivariogram of the grids of estimated area V and raingauge x_i (mm^2), λ_i is the weight of each raingauge, σ_K^2 is the Kriging estimated error variance (mm^2), and μ denotes Lagrange multipliers (mm^2).

The estimated area V must be divided into M grids before calculating hourly mean rainfall of storm events over a watershed by applying Eq. 8. This equation can be rewritten as

$$\begin{cases} \sum_{j=1}^{n}\lambda_j\gamma(x_i, x_j) + \mu = \frac{1}{M}\sum_{m=1}^{M}\gamma(V_m, x_i), & i = 1, 2, \ldots, n \\ \sum_{i=1}^{n}\lambda_i = 1. \end{cases} \tag{10}$$

where V_m is the m-th grid in the estimated area. The block Kriging method has Best Linear Unbiased Estimation (BLUE) geostatistic characteristics. The estimator Z_K^* of hourly mean rainfall is a linear combination of n available point-rainfall recordings $Z(x_i)$, which is located at x_i and with weightsλ_i. The estimator Z_K^* can be expressed as

$$Z_K^* = \sum_{i=1}^{n} \lambda_i Z(x_i) \tag{11}$$

Hourly semivariograms must be determined in advance to compute hour mean rainfall. This study uses the following procedures to calculate these hourly semivariograms $\gamma(t,h_{ij})$:

1 Calculate the values of variance $s^2(t)$ and mean $m(t)$ of rainfall for n rainguages in each time period t. Denote the variances $s^2(t)$ and mean $m(t)$, respectively, as

$$s^2(t) = \frac{1}{n} \sum_{i=1}^{n} [p(t,x_i) - m(t)]^2 \tag{12}$$

$$m(t) = \frac{1}{n} \sum_{i=1}^{n} p(t,x_i) \tag{13}$$

2. Apply Eq. 6 to compute the scaled climatological mean semivariogram $\gamma_d^*(h_{ij}, a)$ of all rainfall events, and use the power model to obtain parameters ω_0 and a.
3. The hourly semivariogram can be obtained using Eq. 5, namely the variance $s^2(t)$ multiplied by the scaled climatological mean semivariogram $\gamma_d^*(h_{ij}, a)$.

The block Kriging method is generally considered appropriate for estimating mean rainfall. When applied to a lumped/distributed model, the block Kriging method can easily calculate the mean rainfall of an entire watershed and its divisions, using the following procedures:

1. Divide the study watershed into a suitable grid. A denser grid can produce a smaller Kriging estimated error variance, but it also increases the computation frequency. Therefore, a suitable grid should minimize the Kriging estimated error variance and computation frequency.
2. Calculate the semivariograms for arbitrary rainguages x_i and x_j, that is the left-hand $\gamma(x_i, x_j)$ of the equal sign in Eq. 10.
3. Calculate the mean semivariogram between the grids of the estimated area V and each rainguage x_i, that is the right-hand $\sum_{m=1}^{M} \gamma(V_m, x_i)/M$ of the equal sign of Eq. 10. Figure 1 shows the computation procedure of the mean semivariogram.
4. Solve the matrix of Eq. 10 to obtain rainguage weightings and then apply Eq. 11 to calculate the hourly mean rainfall and its estimated error variance (Delhomme 1978).

3 The Study Watershed

3.1 Geographical Features

This study uses the Tanshui River Basin as the research area. The Tanshui River is the third longest river in Taiwan (Fig. 2), and its basin covers the Taipei Pan through the Da-Han Stream, Hsin-Tien Stream, and Kee-Lung River. Taipei City and Taipei County, which stand on the Taipei Pan, are home to over 5 million.

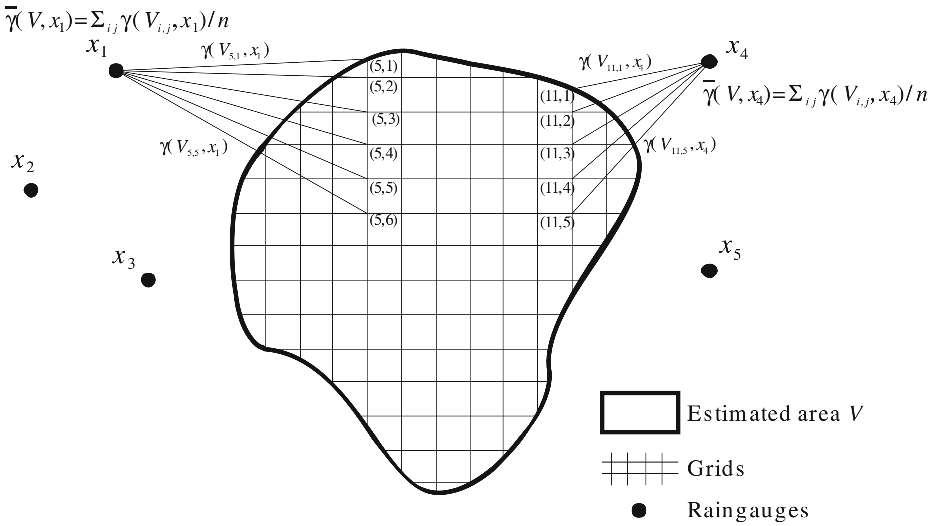

Fig. 1 Computation of the mean semivariogram between the estimated area and rainguages

The Tanshui River system is 159 km long and has a drainage area of 2,726 km^2. Its mean annual precipitation and runoff depth are 3,001 mm and 2,584 mm, respectively. Rainfall is non-uniform in both time and space. Due to the low elevation of the Pan, the stream bottom is well below sea level. Thus, ocean tides typically increase the water level of downstream

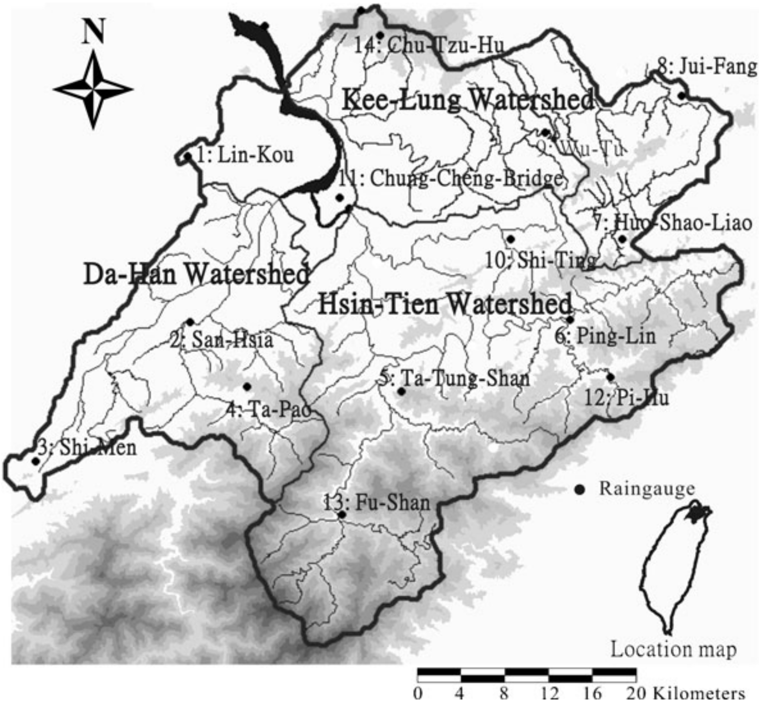

Fig. 2 The location map of 14 telemeter raingauges in Tamshui River Basin

sections of the Tanshui River. During summer, typhoons dump a large quantity of stormwater onto the Taipei Pan, which usually causes massive disasters due to a short flood lead time. Therefore, studying the spatial rainfall distribution of the Tanshui River Basin is extremely important for flood control in this area.

3.2 Study Data

The Tanshui River Basin has 14 telemetry-equipped raingauges. During the watershed study, 1,190 rainfall events occurred between 1966 and 2005. Typhoons accounted for 125 events; the others were large storms. From all the events, 78 have complete recordings from all the raingauges; of these fully recored events, 23 were typhoons and 55 were large storms. In the northern hemisphere, a typhoon is known as a tropical cyclonic with a low-pressure center, long rainfall duration and intensity typically has a large diameter. A storm is convective precipitation or orographic precipitation; the rainfall intensity of a large storm is similar to that of a typhoon, the differences are the diameter and duration. Storm rainfall in a small area may result in few raingauges having a recording of the rainfall depth and several gauges having no data despite the raingauges working well. Therefore, 12 significant typhoons that have complete rainfall recordings, and that each caused flooding disasters, were used in this study. These typhoons are as follows: MURY (1981-07-19), NELSON (1985-08-22), WAYNE (1986-08-22), ABBY (1986-09-17), LYNN (1987-10-23), DOUG (1994-08-07), WINNIE (1997-08-17), ZEB (1998-10-15), XANGSANE (2000-10-30), HAIMA (2004-09-10), MATSA (2005-08-04) and DAMREY (2005-09-20).

4 Results and Discussions

This study uses the Thiessen polygon and block Kriging methods to identify the effects of raingauge distribution and estimate average basin rainfall and representative sites based on their identification. These methods assume that the rainfall depth in a space distribution is a two-dimensional random field constructed using the climatological mean semivariogram. Moreover, each gauge was removed from the original raingauge network in turn. This procedure was used to identify the effects of raingauges and relatively important sites on the watershed.

4.1 The Homogeneity of the Variogram Parameters

An hourly semivariogram is a function of time period t, isotropy, and a time average form with nonzero and T time interval. As previously mentioned, typhoons and storms have differing spatial characteristics due to their different generation mechanisms. To test the heterogeneity between typhoons and storms, two variogram types (global and scaled semivariograms) were separately computed using the rainfall events in three classifications (all events, typhoons, and storms). A global variogram is a semivarogram computated without using $s(t)$, while a scaled variogram is a semivariogram computation using $s(t)$, as shown in Eq. 6 of this study. The global variogram computations using the three classifications of all events, typhoons, and storms were plotted in Fig. 3a. Figure 3b shows the scaled variograms of the three classifications of event types. Based on Fig. 3a, typhoons and storms do not have the same spatial characteristics. The global variogram, with all events, lies between the global variogram of typhoons and that of storms. In Fig. 3b, the three scaled variograms are close together, except for the points in the last interval between

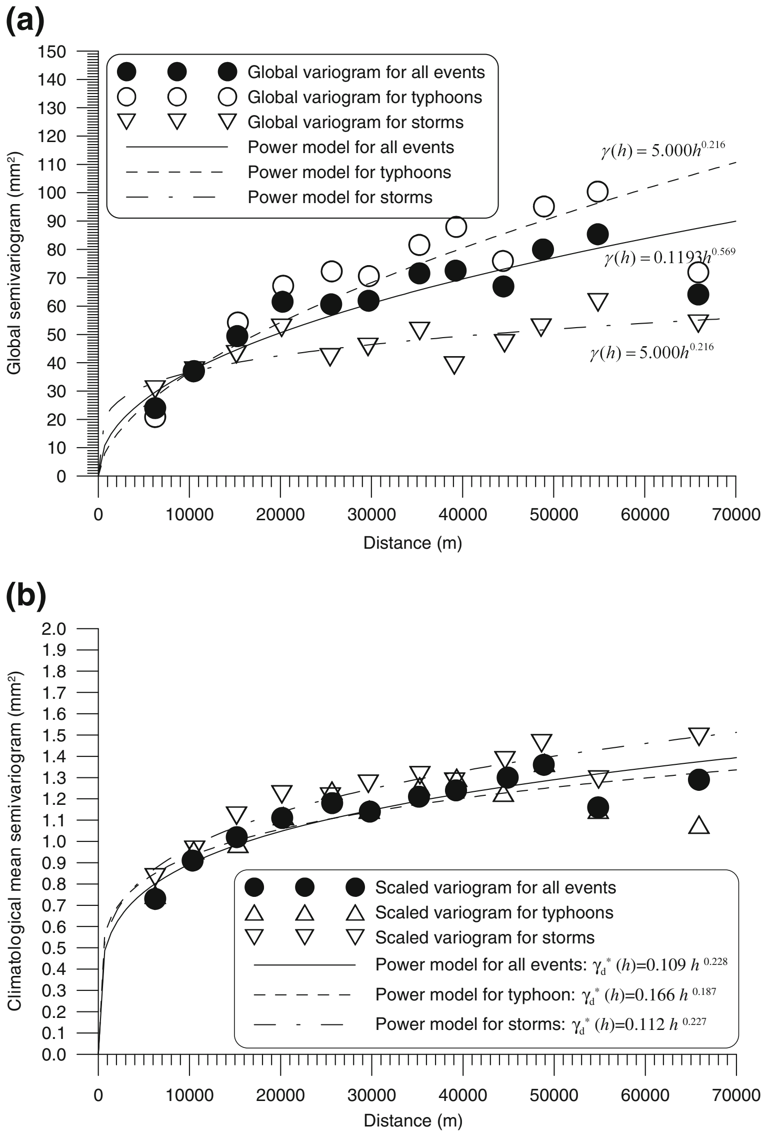

Fig. 3 The global and scaled semivariograms using three classifications of all events, typhoons, and storms

the relative distances of 60000 and 70000 m. These results verify that using a scaled variogram can eliminate the heterogeneity of mixed rainfall data.

4.2 Areal Rainfall Computations

Since using a scaled variogram can eliminate the heterogeneity between typhoons and storms, this study uses all the events, including typhoons and storms, to establish a basic semivariogram of the Tanshui River Basin. Computations for the scaled climatological mean semivariogram of rainfall events recorded by the 14 raingauges in the watershed were completed. The power form, expressed as Eq. 7, was then applied to the basic experimental semivariogram (Eq. 14).

$$\gamma_d^*\left(h_{ij}, a\right) = \omega_0 h^a = 0.109h^{0.228}, \mathrm{R}^2 = 0.873 \tag{14}$$

The variance $s^2(t)$ of a realization $\pi(t)$ for each time period t can be calculated easily from hourly rainfall measurements. The hourly semivariograms of rainfall events can be calculated directly using Eqs. 5 and 14. The estimated area was divided into $M = 2665 \times 1\mathrm{km}^2$ grids to calculate the hourly mean rainfall of rainfall events during the watershed using Eq. 10. This study used observations from 14 raingauges between 1966 and 2005.

4.3 Weight Distribution Changes Due to Missing Data from Raingauges

This study uses observations from 14 telemeter raingauges to estimate the hourly mean rainfall on the Tanshui River Basin for 12 typhoons. To simplify the analyses of the spatial variation of rainfall in the basin, this study only considers parameter a of the semivariogram; this parameter only differs from location to location (Bastin et al. 1984). The weights of the 14 telemeter raingauges were calculated using the Thiessen polygon and block Kriging methods combined with a power semivariogram model (first row in Tables 1 and 2).

This study applies an iterative method (Cheng et al. 2007) to evaluate the weight redistribution of raingauges with missing rainfall data and identify weight variations due to changes in the raingauge distribution. This iterative method removed raingauges from the original raingauge network (n=14) once at a time. The Thiessen polygon and block Kriging methods then calculated the changes in weights for the remaining sites (n=13). Table 1 shows the weight of each site obtained using the Thiessen polygon method. All the same cases were repeated to estimate the spatial rainfall distribution using the block Kriging method. Table 2 shows the computed results based on the block Kriging method. The * notation in Tables 1 and 2 indicates that a raingauge is assumed unknown.

Estimations using the block Kriging and Thiessen polygon methods for hourly mean rainfall were obtained under an unbiased condition. In this unbiased condition, all raingauges have a role in ensuring that the sum of weights equals one. Therefore, if any data from an arbitrary raingauge are missing, the original weight of that raingauge must be assigned to the remaining raingauges. Rows 2 to 15 of Tables 1 and 2 list the redistributed weights. When data from a raingauge is missing, the weights of neighboring stations were changed using the Thiessen polygon method. However, the weights of sites farther away remained unchanged (Table 1). When using the block Kriging method, the weights of the remaining 13 raingauges were higher than the estimated weights of the 14 raingauges with complete data. This shows that when rainfall data from any raingauge is missing, the weights of all existing raingauges must be redistributed using the block Kriging method with a semivariogram model. However, the Thiessen polygon method simply changes the weights of neighboring raingauges based on changes to the polygon network.

The weight of each existing raingauge does not increase uniformly. The increases in weights are higher for raingauges near a raingauge missing data than those far away (Table 2). For instance, when data from raingauge 5 were missing, the weights of neighboring sites (gauges 4, 10, 11, 12, and 13), increased more than those of distant raingauges (gauges 1, 2, 3, 6, 7, 8, 9, and 14). Comparison results for weight redistributions confirm that the block Kriging and Thiessen polygon methods can appropriately revise the weights of existing raingauges. However, redistribution results of these two methods differ.

Table 1 Weighting values at each raingauge using the Thiessen polygon method

Rainagauge	1	2	3	4	5	6	7	8	9	10	11	12	13	14
The value of weightings of the raingauges	0.056	0.064	0.021	0.089	0.120	0.040	0.058	0.033	0.087	0.067	0.110	0.063	0.123	0.071
	*	0.070	0.021	0.093	0.120	0.040	0.058	0.033	0.087	0.067	0.141	0.063	0.123	0.086
	0.058	*	0.032	0.141	0.120	0.040	0.058	0.033	0.087	0.067	0.110	0.063	0.123	0.071
	0.056	0.085	*	0.089	0.120	0.040	0.058	0.033	0.087	0.067	0.110	0.063	0.123	0.071
	0.057	0.117	0.021	*	0.135	0.040	0.058	0.033	0.087	0.067	0.125	0.063	0.128	0.071
	0.056	0.064	0.021	0.108	*	0.056	0.058	0.033	0.087	0.081	0.121	0.073	0.171	0.071
	0.056	0.064	0.021	0.089	0.122	*	0.063	0.033	0.087	0.079	0.110	0.083	0.123	0.071
	0.056	0.064	0.021	0.089	0.120	0.062	*	0.038	0.096	0.074	0.110	0.075	0.123	0.071
	0.056	0.064	0.021	0.089	0.120	0.040	0.071	*	0.106	0.067	0.110	0.063	0.123	0.071
	0.056	0.064	0.021	0.089	0.120	0.040	0.066	0.062	*	0.098	0.111	0.063	0.123	0.087
	0.056	0.064	0.021	0.089	0.124	0.067	0.064	0.033	0.104	*	0.122	0.063	0.123	0.071
	0.077	0.064	0.021	0.117	0.129	0.040	0.058	0.033	0.088	0.094	*	0.063	0.123	0.095
	0.056	0.064	0.021	0.089	0.123	0.091	0.065	0.033	0.087	0.067	0.110	*	0.123	0.071
	0.056	0.064	0.021	0.097	0.235	0.040	0.058	0.033	0.087	0.067	0.110	0.063	*	0.071
	0.074	0.064	0.021	0.089	0.120	0.040	0.058	0.033	0.099	0.067	0.150	0.063	0.123	*

The * notation represents the raingauge that is assumed to be unknown

Table 2 Weighting values at each raingauge using the block Kriging method

Rainagauge	1	2	3	4	5	6	7	8	9	10	11	12	13	14
The value of weightings of the raingauges	0.063	0.077	0.108	0.087	0.082	0.056	0.052	0.052	0.056	0.058	0.071	0.064	0.114	0.060
	*	0.088	0.115	0.092	0.085	0.057	0.054	0.055	0.059	0.060	0.081	0.066	0.117	0.070
	0.076	*	0.120	0.106	0.088	0.057	0.053	0.054	0.058	0.060	0.078	0.066	0.120	0.064
	0.077	0.098	*	0.104	0.089	0.059	0.055	0.057	0.059	0.060	0.076	0.069	0.131	0.067
	0.070	0.098	0.119	*	0.094	0.058	0.054	0.054	0.057	0.061	0.078	0.067	0.128	0.063
	0.066	0.083	0.112	0.098	*	0.064	0.056	0.054	0.059	0.065	0.079	0.072	0.130	0.063
	0.064	0.078	0.109	0.088	0.088	*	0.062	0.055	0.059	0.067	0.073	0.079	0.117	0.062
	0.064	0.078	0.109	0.088	0.085	0.065	*	0.061	0.063	0.065	0.073	0.072	0.116	0.063
	0.065	0.078	0.110	0.088	0.084	0.060	0.062	*	0.067	0.062	0.073	0.069	0.116	0.066
	0.065	0.078	0.109	0.088	0.084	0.060	0.060	0.063	*	0.067	0.075	0.067	0.115	0.068
	0.065	0.078	0.109	0.088	0.087	0.065	0.060	0.056	0.065	*	0.077	0.069	0.116	0.064
	0.073	0.084	0.111	0.093	0.089	0.059	0.055	0.055	0.062	0.066	*	0.066	0.117	0.070
	0.064	0.078	0.110	0.089	0.089	0.074	0.062	0.057	0.059	0.063	0.073	*	0.120	0.062
	0.068	0.087	0.123	0.106	0.106	0.063	0.056	0.056	0.058	0.062	0.076	0.075	*	0.063
	0.073	0.080	0.111	0.089	0.085	0.058	0.055	0.059	0.065	0.063	0.081	0.066	0.116	*

The * notation represents the raingauge that is assumed to be unknown.

The block Kriging method has another condition in addition to the unbiased condition; that is, the estimated error variance is minimal. The Thiessen polygon method does not have this condition. Redistributing weight relies on a redrawn map of the study basin. The Thiessen polygon method is unsuitable for application to real-time hydrological simulation and forecasting systems because redistributing weights is time-consuming and inefficient. The Kriging method uses a spatial model of a semivariogram to redistribute weights due to raingauge network changes, whereas the Thiessen method relies on a change of a polygon distribution resulting from a redrawn map. Hence, the Kriging method is theoretically better than the Thiessen method because it has a spatial structure (i.e., semivariogram), while the Thiessen method has a lesser ability to represent the spatial structure of rainfall.

4.4 Comparison of Rainfall Volume Between Thiessen Polygon and Block Kriging

Because the values of hourly mean rainfall estimated by different methods are not the same, they may generate different hyetograph inputs for the rainfall-runoff routing and different simulated hydrographs for the same rainfall-runoff events. This study compares the results of the Thiessen polygon and block Kriging methods using the total volumes of several rainfall events. Twelve previously-described cases that have complete records were selected for comparisons.

A different number of raingauges may introduce unnecessary errors in the hyetograph of an event and rainfall-runoff routing. This study evaluates the differences in rainfall volume when some data are missing. The iterative method used to examine the estimated rainfall volume of 12 typhoons is as follows: (1) Assume that rainfall data of a raingauge are unknown. Then, compute the rainfall volume of the 12 typhoons using data from the remaining 13 raingauges by applying the Thiessen polygon and block Kriging methods. (2) Assume that rainfall data from another raingauge are missing. Then, calculate the rainfall volume of the 12 typhoons in the same manner. (3) Repeat this procedure until data from all raingauges are assumed unknown. (4) Compare the differences in computed rainfall volume via the iterative procedure and values from all 14 raingauges.

Table 3 lists the rainfall volumes of the 12 typhoons derived by applying the Thiessen polygon and iterative methods. The results in Table 3 show the differences in rainfall volumes between volumes computed by the iterative procedure and volumes computed using all 14 raingauges. A comparison of these results indicates that when data for a raingauge are missing, the volume estimation is either larger (positive values for overestimation) or smaller (negative values for underestimation) than the value computed using all 14 raingauges. Based on the goal of flood control, underestimating rainfall volume may cause worse disasters than overestimating rainfall volume. Therefore, this study only discusses the effects of underestimations of rainfall volumes. Underestimation occurs when the data missing from a raingauge is smaller than the estimation using all 14 raingauges. Using the same procedure as the computed results of Table 3, similar comparisons of rainfall volume using the block Kriging method can be conducted for the presentation for Table 4.

To compare underestimations resulting from the Kriging method with those based on the Thiessen method, Table 5 lists the five statistics used to compare rainfall volume. The sum of event volumes is the total rainfall volume of the 12 typhoons; the underestimation count is the frequency of underestimation out of 168 possibilities (12 typhoons by 14 raingauges); the maximum underestimation is maximum value of underestimation in 168 possibilities; the total underestimation is a sum of underestimation for each computed volume and is a depth unit (mm); and the underestimation percentage is a ratio percentage of total underestimation to the sum of event volumes.

Table 3 Differences in Rainfall volumes estimated from the Thiessen polygon method with the iterative method

Typhoons	Missing raingauge number													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
MURY (1981)	6.4	0.0	1.3	−5.6	5.5	−2.0	−0.3	−1.7	6.1	−4.4	6.3	5.8	−1.8	−12.3
NELSON (1985)	10.8	3.2	−0.3	−0.5	−14.0	−8.5	−14.8	2.5	−1.0	18.8	−16.8	13.6	20.1	−5.0
WAYNE (1986)	16.8	11.9	0.0	−11.9	−12.4	−9.5	−11.9	−4.3	1.7	20.2	−19.5	16.4	15.2	−10.1
ABBY (1986)	21.0	9.1	−0.9	−0.5	−37.6	−10.9	−8.9	−1.0	−7.4	20.8	−16.2	23.0	22.6	−17.1
LYNN (1987)	48.6	14.3	−2.2	2.9	37.0	3.7	6.0	5.2	−11.1	19.7	−48.5	−17.0	−99.6	−62.6
DOUG (1994)	3.5	−0.3	0.6	−0.2	−3.5	−4.4	−13.8	2.5	4.0	8.8	4.1	8.4	4.5	−16.2
WINNIE (1997)	17.5	2.1	−0.1	5.7	−20.1	−10.9	−21.5	4.6	1.7	21.4	−15.9	22.3	14.3	−23.7
ZEB (1998)	20.2	11.5	−0.7	−0.7	−18.6	−13.7	−11.1	−0.1	0.9	24.0	−32.7	20.7	18.8	−14.7
XANGSANE (2000)	27.0	8.2	−0.9	−3.4	−28.1	−8.2	5.4	−0.4	−12.3	19.4	−32.9	5.7	50.2	−6.9
HAIMA (2004)	7.1	0.5	3.1	−3.3	13.3	5.3	5.7	1.6	−9.5	−6.9	−6.8	−11.7	−16.6	1.1
MATSA (2005)	9.2	3.1	2.6	−9.9	−11.6	−8.0	−35.0	−2.6	16.8	15.0	−0.3	23.1	22.8	−13.7
DAMREY (2005)	7.5	8.5	−1.1	−8.7	−1.9	−2.5	−24.1	0.4	3.5	9.6	−15.5	10.0	8.0	7.4

Unit: mm

Table 4 Differences in Rainfall volumes estimated from the block Kriging method with the iterative method

Typhoons	Missing raingauge number													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
MURY (1981)	−4.1	−9.2	−0.2	−7.7	2.3	−0.6	8.3	0.2	0.5	−0.5	−0.9	8.9	1.1	−9.8
NELSON (1985)	4.4	8.3	1.4	0.0	−0.4	−6.5	−29.3	4.7	−0.7	11.4	−4.4	3.4	5.3	−7.1
WAYNE (1986)	6.6	9.2	7.3	−3.6	1.3	−2.7	−18.7	−6.5	5.1	14.0	−4.1	5.3	2.9	−17.9
ABBY (1986)	13.6	13.9	5.5	3.6	−15.6	2.5	−10.1	−5.6	−0.2	15.4	0.4	14.0	−6.9	−17.9
LYNN (1987)	23.5	27.6	14.9	1.2	11.2	21.5	13.8	9.1	6.3	25.3	−19.4	3.8	−32.1	−72.9
DOUG (1994)	0.7	3.5	2.0	4.9	1.1	−7.7	−28.6	7.9	2.8	7.7	1.5	4.0	1.0	−16.2
WINNIE (1997)	7.2	16.8	7.3	8.9	−7.9	−6.7	−45.3	6.1	2.8	16.0	−4.4	12.1	0.4	−31.1
ZEB (1998)	7.9	27.2	14.8	5.7	−0.5	−11.9	−29.7	−3.0	4.0	12.4	−12.5	10.4	5.3	−23.4
XANGSANE (2000)	19.8	10.9	3.1	−4.2	−2.6	−4.7	5.5	−3.6	−3.5	11.1	−9.5	4.0	−1.4	−15.4
HAIMA (2004)	9.0	1.8	11.1	2.2	16.2	3.1	1.8	2.8	−5.5	−31.8	−0.8	−2.2	−6.3	3.5
MATSA (2005)	9.6	13.7	−0.9	0.7	11.6	−30.8	−49.5	−7.2	10.2	11.6	4.4	12.6	8.6	−11.5
DAMREY (2005)	3.4	13.2	5.8	9.2	4.9	−2.6	−56.1	−2.3	5.5	2.4	−4.7	2.7	4.1	3.5

Unit: mm

Table 5 Five statistics for underestimation comparisons of rainfall volume using the Thiessen polygon and block Kriging methods

Statistics	Rainfall volume	
	Thiessen	Kriging
Sum of event volumes (mm)	5128.3	5662.0
Underestimation count	84	65
Maximum underestimation (mm)	−99.6	−72.9
Total underestimation (mm)	−1015.8	−801.6
Underestimation percentage (%)	−19.8	−14.2

The - notation represents underestimation of mean rainfall

Table 5 shows that the underestimation frequency for rainfall volume by the Thiessen polygon method is 84 times and 65 times for the block Kriging method. The maximum underestimation for the block Kriging method is −72.9 mm, which is less than the value of −99.6 mm for the Thiessen polygon method. The total amounts of underestimated rainfall volume were −1,015.8 mm and −801.6 mm for the Thiessen polygon and block Kriging methods, respectively. The underestimation percentages were −19.8% for the Thiessen polygon method and −14.2% for the block Kriging method. The maximum underestimation of rainfall volume for the Thiessen polygon method is roughly −13.31% in Table 3 (data from raingauge 13 are unknown for typhoon LYNN), and −10.70% in Table 4 for the block Kriging method (data from raingauge 14 are unknown for typhoon LYNN). These comparisons show that underestimations of rainfall volumes using the block Kriging method are consistently less than those using the Thiessen polygon method. Therefore, the block Kriging method is superior to the Thiessen polygon method in deriving rainfall volume of missing raingauge data.

4.5 Effects of Raingauges on Rainfall Volume

The block Kriging method is best at estimating rainfall volume. However, missing raingauge data frequently occurs and yields a hyetograph misestimate. There misestimates differ when data from each raingauge are missing. That is, missing data from one significant gauge may result in a larger difference, whereas missing data from an unimportant gauge, the estimations approximate to the original value.

This study uses a hyetograph calculated using the block Kriging method to determine which raingauges play most important roles in hyetograph estimations. Using the same standard, rainfall volume was used to evaluate the effects of raingauges. The sum underestimation and average underestimation of each raingauge were evaluated for 12 typhoons by assuming raingauges were unknown in turn. Figure 4 ranks the sum underestimation and average underestimation for rainfall volume measured by each raingauge from maximum to minimum. The sum underestimation is the total underestimation for the 12 typhoons when the mean rainfall volume missing from one gauge is smaller than the value of all raingauges. The average underestimation is a ratio of the sum underestimation to the underestimation frequency and represents the underestimation amount for each underestimation.

Based on the rainfall volume shown in Fig. 4, the first and second maximum sum underestimations occurs for raingauges 7 and 14, respectively. These two raingauges have

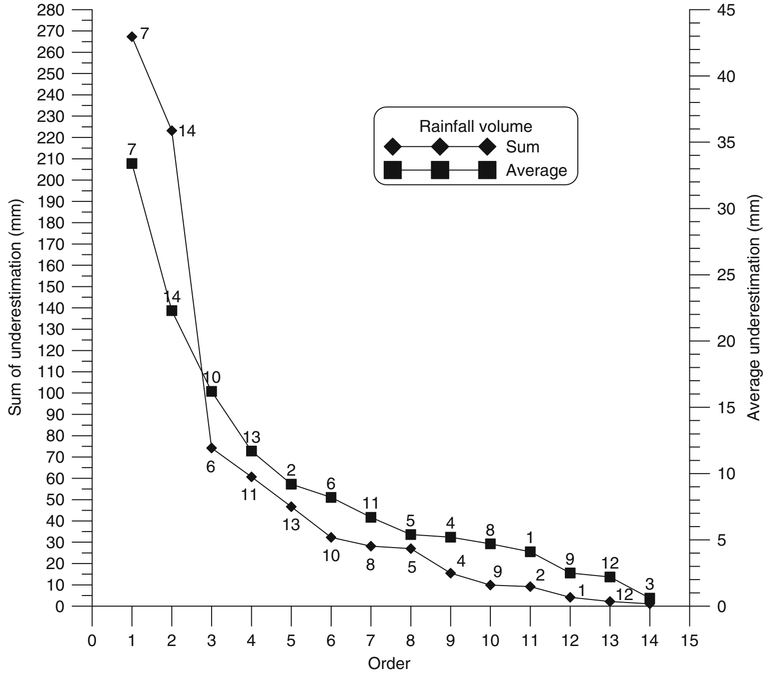

Fig. 4 Raingauge order resulting from sum underestimation and average underestimation for rainfall volume

significantly larger values than those of the other 12 raingauges. Raingauge 6 has the third larger value, followed by raingauges 11, 13, and 10. For an average underestimation, raingauges 7, 14, 10, and 13 have higher values than the other raingauges. Based on the sum and average underestimations, the order of significant raingauges is 7, 14, 6, 10, 13, 11, and 2. These raingauges are more significant than the others for estimating mean hyetographs.

4.6 Site Representativeness in the Raingauge Network

Previous research shows that some underestimations are nearly equal to a common storm volume in Taiwan. The underestimations of rainfall volumes of some typhoons, such as typhoon LYNN (data from raingauge 14 are missing) (Table 4) and typhoons WINNIE and DAMREY (data from raingauge 7 are missing) (Table 4), exceed 1/10 of the total rainfall volume. Identifying the important and dispensable raingauges enables the raingauge network to estimate areal rainfall accurately. Further, the results of this study will help hydrological conceptual models precisely simulate and forecast runoffs. Theoretically, as the number of raingauges increases, the accuracy of a representative estimation of areal rainfall increases. A raingauge network must have at least three raingauges because a plane is composed of at least three points. When analytical results use the two criteria of sum and average underestimations for rainfall volume, raingauges 7, 14, and 6 are the most representative of the raingauge network in the Tanshui River Basin.

Whether three raingauges are sufficient to estimate accurately areal rainfall must be determined. Thus, this study uses raingauges 7, 14, and 6 to estimate the rainfall volume for 12 typhoons and compares these volumes to those computed using all 14 raingauges.

Table 6 lists the comparison results. The rainfall volume derived using three raingauges is larger than that using all 14 raingauges for only one event; the rainfall volume of the other 11 cases is smaller. The underestimation ranges from −15.8% (typhoon MATSA) and −54.3% (typhoon LYNN). These results clearly show that using only three raingauges to estimate mean rainfall is insufficient, and can easily produce large underestimations.

This study uses an approach similar to the iterative method to determine how many raingauges are necessary to estimate areal rainfall of storm events accurately. This procedure is as follows:

1. Identify the order of important raingauges using available criteria. This study uses rainfall volume (Fig. 4).
2. Remove the least important raingauge (e.g., raingauge 3). Estimate the areal rainfall using the remaining 13 raingauges and determine the difference from that using all 14 raingauges.
3. Remove the second least important raingauge. Estimate areal rainfall with the remaining 12 raingauges and determine the difference from that using all 14 raingauges.
4. Continue to remove raingauges one at time. Repeat the procedure above for the raingauges until only three raingauges remain.

Table 7 lists the analytical results for total volume obtained using this procedure. The values in the second column in Table 7 are volume computations for the twelve events using all 14 raingauges. The values in the third column were obtained using 13 raingauges after removing raingauge 3. The values in the fourth column were calculated using 12 raingauges after removing raingauges 3 and 12. To determine raingauge representativeness, the analytical results in Table 7 were recalculated to obtain the sum and average

Table 6 Comparisons of rainfall volumes resulting from three and 14 raingauges using the block Kriging methods

Typhoon events	Rainfall volume			
	V_{all} (mm)	V_3 (mm) (gauges 7, 14, 6)	Difference D (mm)	Difference D (%)
MURY (1981)	335.0	382.7	47.7	14.2
NELSON (1985)	365.2	267.0	−98.2	−26.9
WAYNE (1986)	577.4	380.6	−196.8	−34.1
ABBY (1986)	489.6	253.4	−236.2	−48.2
LYNN (1987)	681.6	311.5	−370.1	−54.3
DOUG (1994)	302.2	231.9	−70.3	−23.3
WINNIE (1997)	392.9	207.1	−185.8	−47.3
ZEB (1998)	611.6	336.7	−274.9	−44.9
XANGSANE (2000)	512.3	319.9	−192.4	−37.6
HAIMA (2004)	564.2	441.2	−123.0	−21.8
MATSA (2005)	570.1	480.0	−90.1	−15.8
DAMREY (2005)	259.3	125.2	−134.7	−51.8

The V_{all} notation represents rainfall volume computed from all 14 raingauges

The V_3 notation represents rainfall volume computed from three raingauges (gauges 7, 14 and 6)

The D (mm) notation denotes the difference of $V_3 - V_{all}$

The D (%) notation is the difference percentage of $V_3 - V_{all}$, which denotes as $\frac{V_3 - V_{all}}{V_{all}} \times 100\%$

underestimations (Fig. 5). Upon removing insignificant raingauges, such as raingauges 3, 12, and 1, the sum and average underestimations were small. The underestimations increased rapidly upon removing significant gauges, such as gauges 14, 6, and 10. Based on these findings, it is possible to determine the representativeness and relative ranking of each raingauge. The representative raingauges are 7, 14, 6, 10, 13, 11, and 2. These results show that the raingauge network in the study watershed must have a minimum of seven raingauges. These representative raingauges must be preserved to reduce underestimation of the mean rainfall during rainfall-runoff processes.

5 Conclusions

This study used the optimal estimation method of areal rainfall to identify the effects of raingauge distribution and representative sites on a watershed. The optimal method is the block Kriging method, which was compared to the results derived from the Thiessen polygon method regarding changes to weighting distributions and volume estimations of average basin rainfall. In the block Kriging method, the climatological mean semivariogram is the product of the scaled climatological mean semivariogram and the sill parameter. The scaled climatological mean semivariogram is time invariant and only depends on raingauge locations, whereas the sill parameter is time variant. The scaled variogram has an ability to eliminate heterogeneity between typhoons and storms. Therefore, a climatological mean semivariogram was utilized to illustrate the temporal-spatial variations of mixed rainfall data (typhoons and storms) and can easily derive the theoretical semivariogram model.

When raingauge distribution changes during a watershed, the Thiessen polygon method only changes the weights of neighboring raingauges, leaving the weights of other raingauges unchanged. However, the block Kriging method redistributes the weights among all raingauges. In these cases, the increased weights are higher for raingauges near a raingauge with missing data. Therefore, the block Kriging method can automatically revise

Table 7 Raingauge representativeness and their relative ranking based on volume computations of areal rainfall for 12 typhoons

Deletion order		1	2	3	4	5	6	7	8	9	10	11
Gauges	All	3	12	1	9	4	5	8	2	11	13	10
MURY (1981)	335.0	321.0	322.1	336.2	336.9	339.9	344.1	350.2	362.3	367.1	389.4	382.7
NELSON (1985)	365.2	361.9	366.8	361.7	359.5	369.9	366.2	368.5	340.4	317.8	299.2	267.0
WAYNE (1986)	577.4	560.1	561.6	570.4	560.0	579.0	580.2	566.7	526.2	484.6	439.5	380.6
ABBY (1986)	489.6	481.4	468.7	493.8	483.4	494.0	477.7	457.4	447.6	417.0	298.2	253.4
LYNN (1987)	681.6	613.8	563.2	547.1	491.2	504.1	462.7	442.9	435.8	426.5	364.7	311.5
DOUG (1994)	302.2	287.7	287.0	278.0	287.8	291.1	297.7	309.1	273.8	252.1	236.4	231.9
WINNIE (1997)	392.9	366.4	362.8	367.3	366.7	372.1	364.2	361.9	312.3	277.4	215.1	207.1
ZEB (1998)	611.6	595.0	598.5	608.2	586.4	598.1	574.2	554.2	510.5	452.4	375.5	336.7
XANGSANE (2000)	512.3	505.4	506.3	499.1	475.3	490.4	463.3	445.6	449.6	412.8	363.3	319.9
HAIMA (2004)	564.2	562.6	553.4	544.7	538.6	497.8	473.3	471.7	476.8	474.3	469.0	441.2
MATSA (2005)	570.1	557.8	566.9	563.3	580.4	593.0	609.8	605.6	545.4	498.5	507.3	480.0
DAMREY (2005)	259.9	256.5	260.7	255.8	263.8	255.7	262.9	259.9	177.2	157.5	153.0	125.2

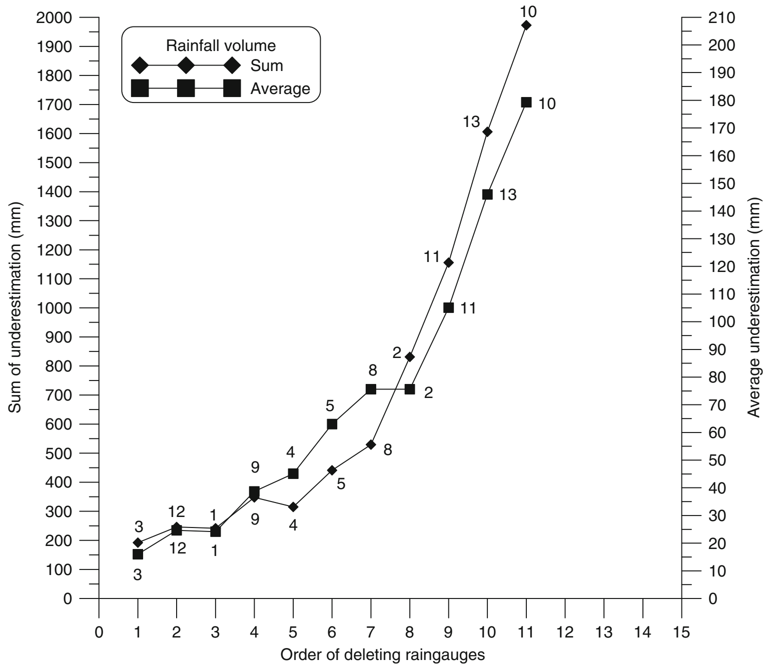

Fig. 5 Raingauge representativeness of the network in the Tamshui River Basin

and suitably weight the remaining raingauges and estimate the hourly mean rainfall. The block Kriging method is more efficient than traditional methods, which only redraw the watershed map. Traditional methods cannot rapidly manage the weights of raingauges when data are missing. Therefore, the Kriging method is theoretically super to the Thiessen method because the Kriging method has a spatial structure (semivariogram), while the Thiessen method has a lesser ability to represent the spatial structure of rainfall. When the raingauge network changes due to missing data, the block Kriging method is suitable for estimating the spatiotemporal rainfall distribution.

When data from a raingauge are missing, the possibility of underestimating rainfall volume increases and may result in serious weather disasters. The maximum underestimation of rainfall volume is approximately 100 mm; the underestimation percentage exceeds 10% in this study. Hydrologists should consider the possibility of underestimating flooding. Comparisons of sum underestimation and underestimation percentages show that underestimations using the block Kriging method are consistently less than those using the Thiessen polygon method are, demonstrating that the block Kriging method is superior to the Thiessen polygon method.

To analyze the representativeness of raingauges in estimating the rainfall volumes of storm events, each raingauge in the raingauge network was sorted from maximum to minimum based on the sum and average underestimations for the estimated rainfall volume. The order of the significant raingauges for estimating rainfall volume was 7, 14, 6, 10, 13, 11, and 2. These raingauges enable the network to estimate areal rainfall accurately. The raingauge network in the study watershed requires at least seven raingauges to produce accurate estimates, and these representative raingauges should be retained to reduce the underestimation of mean rainfall during rainfall-runoff processes.

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