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Influence of heterogeneity on unsaturated hydraulic properties: (1) local heterogeneity and scale effect

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Abstract:

Spatial heterogeneity is ubiquitous in nature, which may significantly affect the soil hydraulic property curves. The models of a closed-form functional relationship of soil hydraulic property curves (e.g. VG model or exponential model) are valid at point or local scale based on a point-scale hydrological process, but how do scale effects of heterogeneity have an influence on the parameters of these models when the models are used in a larger scale process? This paper uses a two-dimensional variably saturated flow and solute transport finite element model (VSAFT2) to simulate variations of pressure and moisture content in the soil flume under a constant head boundary condition. By changing different numerical simulation block sizes, a quantitative evaluation of parameter variations in the VG model, resulting from the scale effects, is presented. Results show that the parameters of soil hydraulic properties are independent of scale in homogeneous media. Parameters of α and n in homogeneous media, which are estimated by using the unsaturated hydraulic conductivity curve (UHC) or the soil water retention curve (WRC), are identical. Variations of local heterogeneities strongly affect the soil hydraulic properties, and the scale affects the results of the parameter estimations when numerical experiments are conducted. Furthermore, the discrepancy of each curve becomes considerable when moisture content becomes closer to a dry situation. Parameters estimated by UHC are totally different from the ones estimated by WRC.

KEY WORDS scale effect; local heterogeneity; soil hydraulic property; VSAFT2

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INTRODUCTION

Soil is a three-phase heterogeneous media. As the volume of a sample of this media increases, a volume is reached where the bulk macroscopic properties become approximately uniform. This is called the relative elemental volume (REV), and Bouma (1984) has suggested that this occurs when the length scale of the sample is approximately 20 times greater than the basic structural unit. This structural unit is the largest grain size in single grained materials and the largest aggregate in aggregated materials. The smaller spatial scale level of the textural information within its structural units is distributed either deterministically (Gardner, 1958; Mualem, 1976; Stephens and Rehfeldt, 1985; Marison *et al*., 1994; Sauer and Logsdon, 2002) or stochastically (Yeh *et al*., 1985a, 1985b, 1985c; Hopmans and Stricker, 1989; Yeh, 1989), for example, using scaling of soil hydraulic properties from laboratory soil cores (Yeh and Harvey, 1990; Eching and Hopmans, 1993; Wildenschild and Jensen, 1999a). The upscaling from the textural to the structural scale level may result in effective, scale-appropriate soil hydraulic functions that may differ in form and parameter values between scales but serve a similar function across scales (Hopmans *et al*., 2002).

Distributed modeling in heterogeneous porous media involves a physically based point model and attempts to incorporate smaller-scale data (e.g. point data) into larger scale modeling by using the REA (or REV), over which, the parameter values are either constant or where the effects of subgrid heterogeneity are parameterized (Albert, 2000). Because of the reason of the typical nonlinearity of physical properties (e.g. water retention curve [WRC] and relative permeability), the scale effects of heterogeneity are inherently problematic.

In unsaturated flow hydrological modeling, the closed-form functions play an important role by indirectly quantifying unsaturated hydraulic data using soil properties that can already or easily be estimated (van Genuchten *et al*., 1991; Marison *et al*., 1994). Many works of upscaling either unsaturated hydraulic conductivity curve (UHC) or WRC have been discussed in the past. Green *et al*. (1996) conducted numerical simulations and upscaled WRC using van Genuchten's (VG) function for both local and upscaled WRC characteristics. They concluded that the V.G. function is useful for modeling WRC at large scales. Desbarats (1995, 1998) addressed the problem of upscaling WRC in randomly heterogeneous porous media and used the method of Leverett

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(1941) to normalize the local WRC. Mantoglou and Gelhar (1987) investigated the method of upscaling UHC by simplifying WRC without physical justification. Chen *et al.* (1994) addressed the method of upscaling UHC under the assumption that heterogeneity is based solely on variations in saturated hydraulic conductivity.

For practical purposes, parameters of soil hydraulic property can be estimated by empirical models of h-$\theta(h)$ or h-$K(h)$ relationship using nonlinear curve fitting with water retention data or unsaturated soil hydraulic data collected from a limited number of *in situ* field measurements or analysis of small soil cores in the laboratory (Yeh and Harvey, 1990; Wildenschild and Jensen, 1999a, 1999b). Notwithstanding, WRCs have been studied extensively at the core scale, whereas application of the WRC models to heterogeneous porous media requires further analysis. Because the scale of heterogeneity often is smaller than the scale of computational grid size, spatial averaging or upscaling is required (Green *et al.*, 1996). Most of the uncertainty of the assessment of water in unsaturated soil at the field scale can be attributed to soil spatial variability caused by soil heterogeneity. The dependence of both soil water retention and unsaturated hydraulic conductivity with moisture content differs among soil types with different particle size compositions and pore size geometry within a heterogeneous media.

As has been mentioned by Hopmans *et al.* (2002), the scaling problem cannot be solved by simple consideration of the differences in space or time scale for several reasons. First, spatial and temporal variability in soil hydrological properties create uncertainties with changing scales. Second, flow and transport processes in vadose zone hydrology are highly nonlinear. To determine the influence of stones on hydraulic conductivity of saturated soils, Sauer and Logsdon (2002) identified, using infiltration tests, a small increase of effective saturated hydraulic conductivity (K_{se}) with an increase of the stone content in two soil types at the pressure head of 12 *cm*. They estimated the relationship of K_{se} with the ratio of the volumetric stone and the soil volume using infiltration tests, and their results were likely be explained by spatial variability of both variables. Novák *et al.* (2011) used a two-dimensional simulation model to show the development of unsaturated zones underneath the stones depending on their sizes and shapes. Wu *et al.* (2011) used numerical experiments to examine the influence of heterogeneity on hydraulic parameters of the van Genuchten model. Their results showed that soil hydraulic properties are strongly affected by variations of heterogeneities and their arrangements. They concluded that the parameters estimated from both WRC and UHC are affected by patterns of heterogeneity; this indicates that the parameters obtained from the WRC are not suitable for predicting the UHC in different patterns of heterogeneous media. Figure 1 illustrates the concept of the scale effects in heterogeneous subsurface porous media. The models of a closed-form functional relationship of soil hydraulic property curves (e.g. V.G. model or exponential model) are valid at point or local scale based on a point-scale hydrological process, but how do scale effects of heterogeneity have an influence on the parameters of these models when the models are used in a larger scale process? Many upscaling approaches, which are mainly averaging schemes, have been used to characterize spatial heterogeneity and develop a methodology to estimate its representative parameters of soil hydraulic property in a larger scale process (Khaleel *et al.*, 2002; Zhu *et al.*, 2004, 2007). However, the averaging process estimated from discrete, small-scale samples may not describe the true soil physical behavior involving a larger spatial structure.

The purpose of this paper is to conduct numerical experiments for the following: first, examining the influence of heterogeneity on soil hydraulic properties of the conventional closed-form functional relationships (e.g. V. G. model or exponential model); and second, investigating the setup of the scale effect of different simulation sizes for numerical experiments on soil hydraulic properties of these models. The results of numerical experiments are discussed accordingly.

METHODOLOGY

Flow through variably saturated porous media

For heterogeneous isotropic porous media under a variably saturated condition, the governing equation takes the following form (Yeh *et al.*, 1993; Khaleel *et al.*, 2002)

$$\frac{\partial}{\partial x}\left[K(h,x,y,z)\frac{\partial h}{\partial x}\right]+\frac{\partial}{\partial y}\left[K(h,x,y,z)\frac{\partial h}{\partial y}\right] +\frac{\partial}{\partial z}\left[K(h,x,y,z)\left(\frac{\partial h}{\partial z}+1\right)\right] = [C(h,x,y,z)+\beta S_s(x,y,z)]\frac{\partial h}{\partial t} \tag{1}$$

where h is the pressure [L] and is positive if the medium is fully saturated or negative if the medium is unsaturated; $K(h,x,y,z)$ is unsaturated hydraulic conductivity [L/T] values in the x, y, and z directions; $C(h)=\frac{\partial\theta}{\partial h}$ is the moisture capacity [1/L]; $\theta(h)$is the volumetric water content, a function of the pressure head; β is the saturation index used to control the storage property of the medium; and S_s is the specific storage of the porous medium [1/L]. Equation (1) is more realistic and flexible than the classical approaches that partition a geologic medium into unsaturated and saturated zones and employs different equations for the processes in different zones. It is applicable to either homogeneous or heterogeneous media. If the medium is homogeneous (i.e. spatially invariant constitutive relations), the unsaturated hydraulic conductivity can vary in time and space because of its dependence on pressure and moisture content (Yeh *et al.*, 1993, 2005). On the other hand, if the medium is heterogeneous, the parameters (e.g. K_s, α, m, n, θ_s and θ_r) in the WRC and UHC are functions of spatial coordinates. In the VSAFT2 model (Yeh *et al.*, 1993), a saturation index in Equation (1), β is used to control the storage property of the medium – for fully saturated media, β is set to one; otherwise its value is zero.

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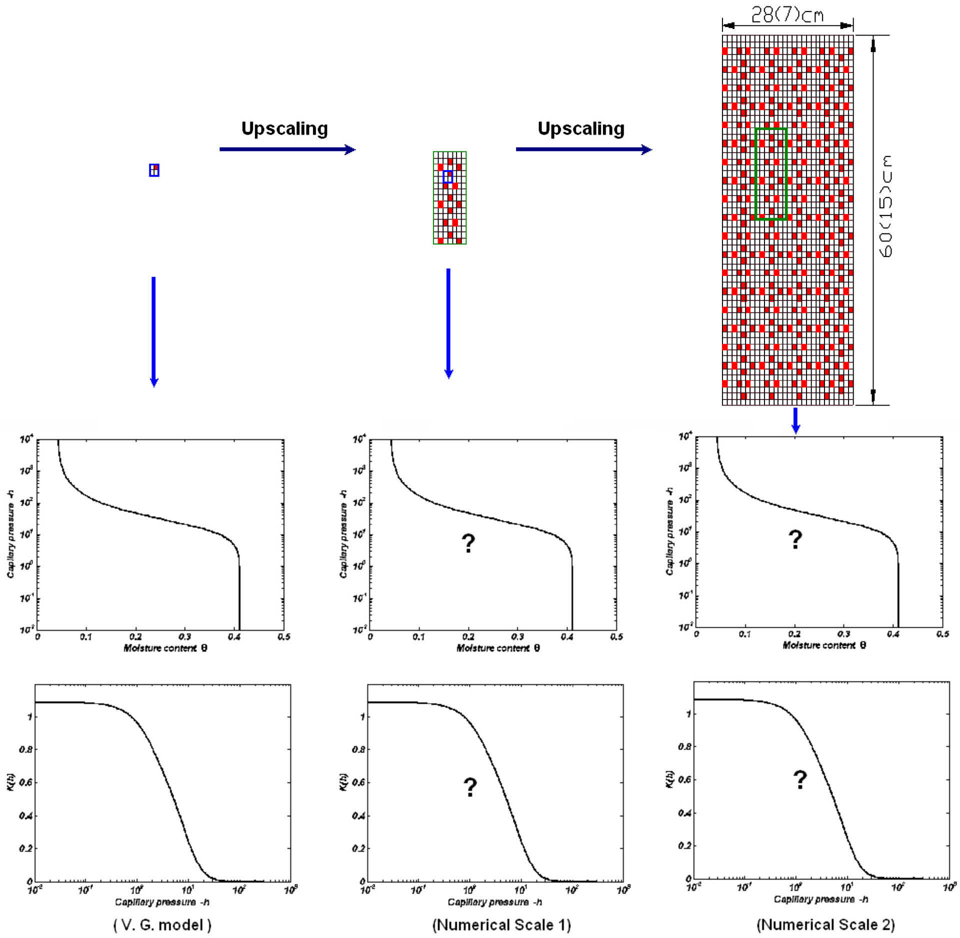

Figure 1. Scale effects of heterogeneous subsurface porous media

Soil hydraulic property models

The soil WRC and UHC are two important soil hydraulic property curves. The soil WRC defines the moisture content as a function of the pressure head, and the UHC establishes a relationship between hydraulic conductivities and the pressure head or moisture content.

For many practical purposes and convenience, mathematical models of closed-form expressions often are used to describe this relationship. One formula, frequently used to depict the unsaturated hydraulic conductivity and WRCs, is the **exponential model** (Gardner, 1958):

$$K(h) = K_s \exp(\alpha h) \tag{2}$$

$$\theta(h) = (\theta_s - \theta_r)\exp(\alpha h) + \theta_r \tag{3}$$

where K_s is the saturated hydraulic conductivity; α is the pore-size distribution parameter; $[1/L]$ represents the rate of reduction in conductivity as the soil desaturates; θ_s is the saturated moisture content; and θ_r is the residual moisture content.

By rearranging Equation (3), we obtain the following relationship:

$$K(h) = A\theta(h) + B \tag{4}$$

where $A = \frac{K_s}{\theta_s - \theta_r}$ and $B = \frac{K_s\theta_r}{\theta_s - \theta_r}$; A and B are constants if K_s, θ_s, and θ_r are known. Equation (4) reveals the linear relationship between $K(h)$ and $\theta(h)$, but this relationship only holds under the assumption that parameter α is the same in both Equations (2) and (3). For homogeneous media, Equation (4) is true, but in heterogeneous media, it may be questioned. The exponential model has been very popular owing to its simplicity and convenience in making mathematical analyses. However, it fits the observed $K(h)$ or $\theta(h)$ data only over a limited range of pressure head values.

The other widely used model for $K(h)$ and $\theta(h)$ is the van Genuchten model (**VG model**) (Mualem, 1976; van Genuchten, 1980):

$$K(h) = K_s \frac{\left(1\text{-}(\alpha|h|)^{n-1}\left[1 + (\alpha|h|)^n\right]^{-m}\right)^2}{\left[1 + (\alpha|h|)^n\right]^{m/2}} \tag{5}$$

$$\theta(h) = (\theta_s - \theta_r)\left[1 + (\alpha|h|)^n\right]^{-m} + \theta_r \tag{6}$$

where α $[1/L]$, n [ ], and m [ ] are soil parameters and $m = 1 - 1/n$. Unlike the exponential model, the relationship between $K(h)$ of Equation (5) and $\theta(h)$ of Equation (6) is nonlinear. Parameters α, n, and m values of unsaturated hydraulic conductivity and moisture release curves often are conveniently assumed to be the same, although they may be different (e.g. Yeh and Harvey, 1990). This model is valid over a broader range of pressure values than the exponential model (van Genuchten and Nielsen, 1985). Because of the use of these mathematical models for the functional relation between the unsaturated hydraulic conductivity, pressure head, and moisture content, soils often can be categorized by parameters such as α, n, θ_s, θ_r, and K_s. For example, coarse-textured soils are reported to have large values of α, n, and K_s, and fine-textured soils are reported to have small values (Table I) (e.g. Stephens and Rehfeldt, 1985). However, values of these parameters are not necessarily unique for a given geological medium because of the hysteretic behavior in the $K(h)$ and $\theta(h)$ relationship; these values can be different according to the wetting and drying histories of the medium.

For the VSAFT2, various soil hydraulic property models such as the exponential model, VG model, and user specified model may be selected for describing the relationship of the pressure head with moisture content and unsaturated hydraulic conductivity. In this paper, the hysteresis effect is ignored to simplify the influent factors of soil physics.

BLOCK SIZE, GRID SIZE, AND SCALE

There are many different aspects of the term 'scale', and sometimes, they are inconsistent with each other. When one refers to the term scale, one generally means the space scale such as the flume length in a laboratory or the dimension of an aquifer in a field. It should be pointed out that scales are physically created from samplings or measurements.

Three terms, related to scale and size, are defined in the context as: (i) block size, (ii) grid size, and (iii) scale. 'Block size' means the dimension of the experimental flume $H \times W$ (e.g. 15 x 7 *cm*, 30 x 14 *cm*), 'grid size' denotes the discrete size for conducting numerical calculations (1 x 1 *cm* in this paper for all experimental cases), and 'scale' here is the ratio between different simulation block sizes (e.g. x 1, x 4, x 16, x 64). We carried out a simulation of a block size 15 x 7 *cm* with a grid size 1 x 1 *cm*. Therefore, the parameters of the soil hydraulic property are based on the scale of a 15 x 7 *cm* block. The block size 15 x 7 *cm*, a usual volume that is used for determining basic soil texture and soil hydraulic characteristics (Novák *et al.*, 2011), was applied to the numerical experiment. The scale then was enlarged four times (x 4), sixteen times (x 16), and thirty-six times (x 36) to investigate variations of parameters for hydraulic properties.

NUMERICAL EXPERIMENTS

Numerical experiments were conducted to examine the influences of local heterogeneities and scale effects on soil hydraulic properties in subsurface porous media. A quantitative evaluation of the parameter variations in the VG and exponential models that resulted from the scale effects was presented by changing different numerical simulation block sizes under both homogeneous and heterogeneous soil flumes. A two-dimensional variably saturated flow and solute transport finite element model (VSAFT2) was used to simulate variations of pressure and moisture content in the soil flume under a constant head boundary condition. Under given boundary conditions, the moisture content and specific flux were obtained at each grid point, and then, the mean moisture content under specific pressure heads of the larger simulation sizes were calculated. The unsaturated hydraulic conductivity was obtained for the different pressure heads by the soil hydraulic property model.

Table I. Parameters of porous media for numerical experiments

	Parameters					
	Soil type	θ_s	θ_r	K_s (cm/h)	α (cm^{-1})	n
1 x 1 *cm**	Sandy loam	0.412	0.041	1.09	0.0523	1.857
	Clay	0.385	0.180	0.03	0.0270	1.600

*Grid size as defined in Figure 4.

Experimental setup

Synthetic media flumes were simulated using VASFT2, and the steady state flow simulations were carried out on unsaturated porous media. Both homogeneous and heterogeneous soil flumes were simulated to examine the local heterogeneity effect on soil hydraulic properties. The observed data of moisture content $\theta(h)$ *versus* pressure head h and the unsaturated hydraulic conductivity $K(h)$ *versus* pressure head h were obtained through numerical experiments. Accordingly, the parameters of the VG model (α and n) were estimated either from UHC or WRC by minimizing a nonlinear objective function (Wu *et al.*, 2011).

Two types of soil were used for numerical experiments, and their parameters are listed in Table I. The experimental flume contained predominantly sandy loam soil and 23% of embedded clay, which was added to create heterogeneity. The UHC*s* of the VG model for both sandy loam soil and clay soil are shown in Figure 2, and the WRCs of the VG model for both sandy loam soil and clay soil are in Figure 3. In Figure 3, the WRCs of both the sandy loam and clay soils intersect.

To investigate the scale effect on soil hydraulic properties, different simulation block sizes were setup for the numerical experiments. The smallest scale with a dimension

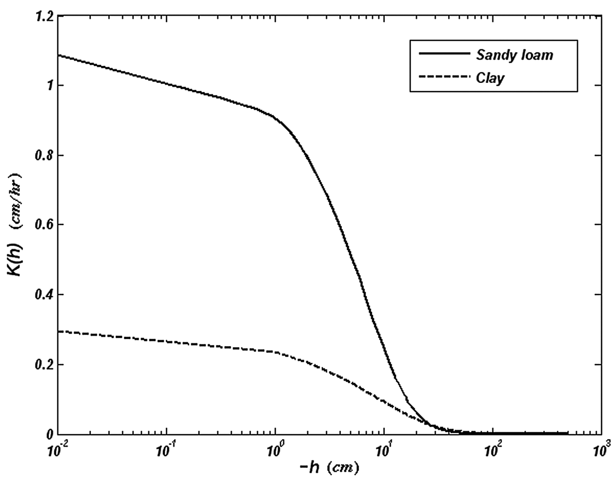

Figure 2. Unsaturated hydraulic conductivities of the VG model for both sandy loam soil and clay soil

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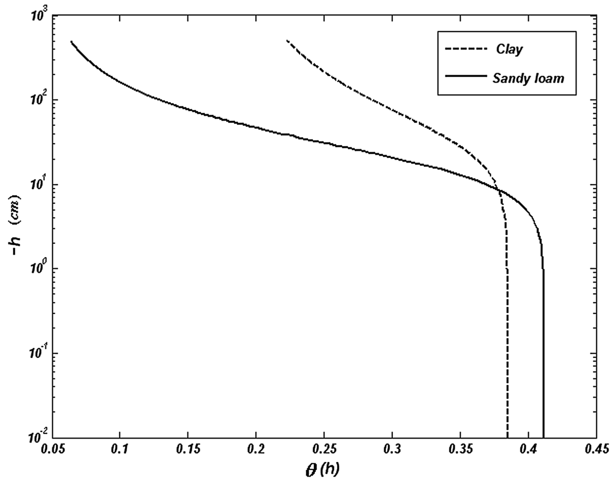

Figure 3. Water retention curves of the VG model for both sandy loam soil and clay soil

of 15 x 7 *cm* ($H \times W$) is shown in Figure 4. The numerical model is simulated with the specific boundary conditions at the top and bottom of the flumes as given in Table II. As can be seen, the same pressure heads are given for the top boundary and bottom boundary. The total head of the upper boundary equals to the head of the lower boundary plus the height (H) of the experiment flume.

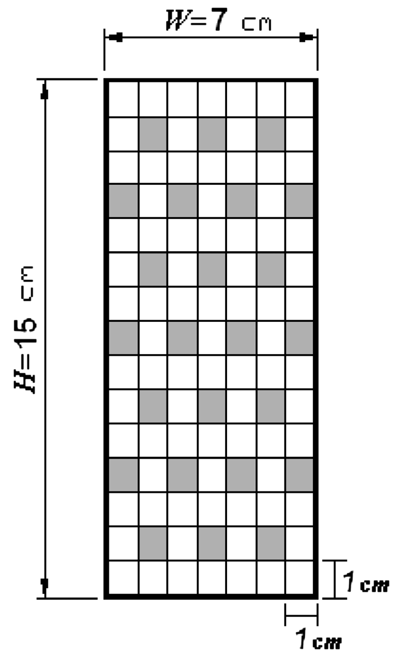

Figure 4. Dimensions of a soil flume (unit block)

Heterogeneity setup

The homogeneous experimental soil flumes are filled with sandy loam where $\theta_s = 0.412$, $\theta_r = 0.041$, $\alpha = 0.0523\,cm^{-1}$, $K_s = 1.09\,cm/h$, $n = 1.857$ for a cell dimension of 1 x 1 *cm*. The heterogeneous experimental soil flume contains homogeneous sandy loam where $\theta_s = 0.412$, $\theta_r = 0.041$, $\alpha = 0.0523\,cm^{-1}$, $K_s = 1.09\,cm/h$, $n = 1.857$ embedded in 24% of local heterogeneity (clay) where $\theta_s = 0.385$, $\theta_r = 0.18$, $\alpha = 0.027\,cm^{-1}$, $K_s = 0.03\,cm/h$, $n = 1.6$ for a cell dimension of 1 x 1 *cm*. Clay blocks with a constant size of 1 × 1 *cm* were embedded as the local heterogeneity (Figure 4). Given parameters are listed in Table I.

Numerical simulation scale

Simulation block size of soil flumes with dimensions of 15 x 7, 30 x 14, 60 x 28, and 90 x 42 *cm* were established for the numerical experiments. Both homogeneous and heterogeneous soil flumes were simulated. Based on the smallest block (15 x 7 *cm*) with simulation grid size 1 x 1 *cm*, the other numerical simulation soil flumes were magnified 4, 16, and 36 times, respectively. Embedding heterogeneity into each magnified soil flume under specific dimensions, the upscaling soil flumes were simulated.

RESULTS AND DISCUSSION

Two themes of numerical experiments were conducted for the following: (i) examining the influence of heterogeneity on soil hydraulic properties of the VG model; and (ii) investigating the scale effect of different simulation sizes for the numerical experiments on the soil hydraulic properties of the VG and exponential models. Results of the numerical experiments are discussed accordingly.

Influences of local heterogeneities on a soil hydraulic property model

Influences of local heterogeneities on a soil hydraulic property model are investigated by carrying out numerical experiments through VSAFT2. Figure 4 shows the smallest simulation block size of a soil flume. Figure 5 presents the fitted parameters of α (0.0523) and n (1.857) for homogeneous soil flumes using WRC. The solid curve is obtained by the VG model, and the circular symbols are the results of the VSAFT2 model. Similar to Figure 5, Figure 6 shows the fitted parameters of α (0.0523) and n (1.857) for homogeneous soil flumes using UHC. Soil hydraulic properties of α and n are exactly the same despite using UHC or WRC (as shown in Tables III and IV). Figure 7

Table II. Boundary conditions for numerical experiments

Exp. no.	1	2	3	4	5	6	7	8	9	10	11	12
Upper B.* (*cm*)	−0.00001	−1	−5	−10	−20	−30	−40	−50	−100	−150	−200	−300
Lower B.* (*cm*)	−0.00001	−1	−5	−10	−20	−30	−40	−50	−100	−150	−200	−300

*Pressure head (*cm*).

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illustrates different numerical block sizes of homogeneous media. Four kinds of numerical simulation blocks are simulated, namely, the 15 x 7, 30 x 14, 60 x 28, and 90 x 42 *cm* blocks. Figure 8 shows the fitted parameters of α and

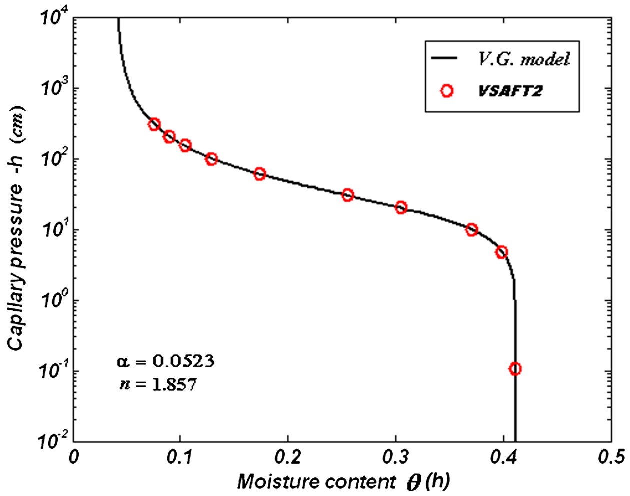

Figure 5. Water retention curve: homogeneous soil flume (fitted by VG model)

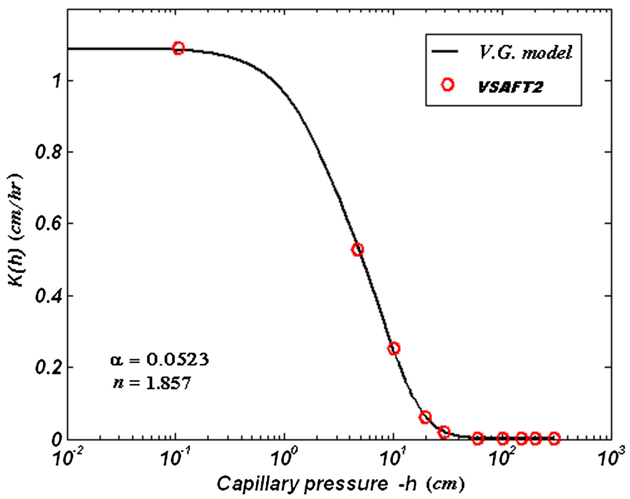

Figure 6. Unsaturated hydraulic conductivity curve homogeneous soil flume (fitted by VG model)

n for homogeneous soil flumes using UHC under different numerical simulation block sizes. In the figure, the solid line is obtained from the VG model, and the circular, triangular, cross, and square symbols are the results obtained from the dimensions of 15 x 7, 30 x 14, 60 x 28, and 90 x 42 *cm* blocks, respectively. Similarly, Figure 9 presents the fitted parameters of α and *n* for homogeneous soil flumes using WRC under different numerical simulation blocks. As can be seen from both Figures 8 and 9, all the symbols of different scales overlie the solid curve. This demonstrates that the parameters of the soil hydraulic properties are independent of the scales in homogeneous media. Despite tiny fluctuations, the fitted parameters of α and *n* of homogeneous media by using UHC and WRC, respectively, are identical. In other words, for a homogeneous case, the hydraulic properties are the same for both UHC and WRC regardless of the expected scale.

Influences of scale effects on a soil hydraulic property model

Although local heterogeneities were embedded into the homogeneous experimental soil flumes, the parameters of the soil hydraulic property changed significantly. Figure 10 shows the UHCs of both the homogeneous and heterogeneous media. In the figure, the solid line is the curve obtained from the VG model, the circular symbols are simulation results of homogeneous media, and the triangular line is the result of soil flumes that are 24% heterogeneous. As can be seen, the circular symbols overlie the solid curve, whereas the triangular line shows to be quite different from them. When the negative pressure head is higher than a certain value (i.e. $-h > 30\,cm$), all UHCs together overlie each other. This indicates that the soil condition at this specific pressure becomes very dry and results in the relative conductivity to approach zero. On the contrary, when the negative pressure head is low, the soil moisture increases, and the discrepancy of UHC becomes obvious when heterogeneity increases (24%). Similarly, Figure 11 presents WRCs for homogeneous and heterogeneous media. As can be seen, when the negative pressure head is less than a certain value (i.e. $-h < 5\,cm$), each WRC remains constant. The parameter sets α and *n* that are obtained from the

Table III. Fitted parameters using unsaturated hydraulic conductivity [VG model]

Soil flume *cm* x *cm*	Percentage of heterogeneity	α (cm^{-1})	*n*	K_s (*cm/h*)
7 x 15	homogeneous (0%)	0.0523	1.857	1.090
	heterogeneous (24%)	0.0028	0.9275	0.763

Table IV. Fitted parameters using water retention curve [VG model]

Soil flume *cm* x *cm*	Percentage of heterogeneity	α (cm^{-1})	*n*	K_s (*cm/h*)
7 x 15	homogeneous (0%)	0.0523	1.857	1.090
	heterogeneous (24%)	0.3342	1.520	0.830

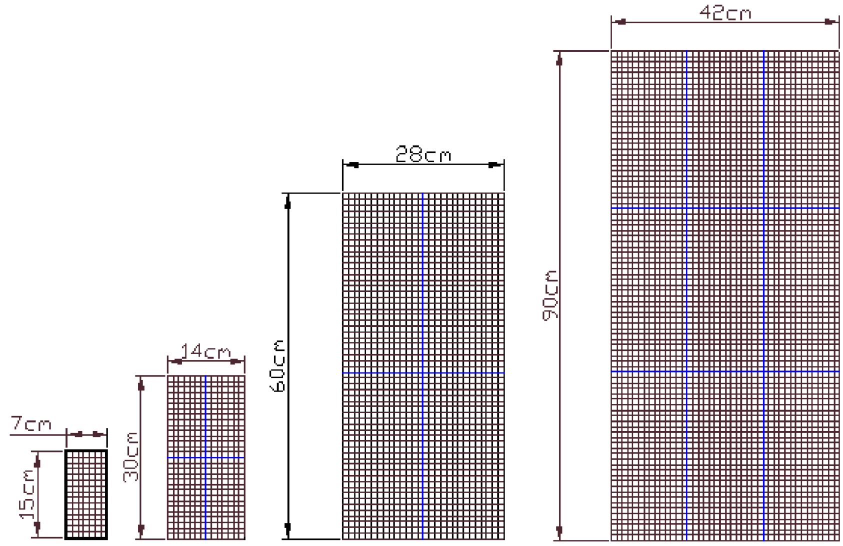

Figure 7. Different numerical grid sizes of soil flumes (homogeneous media)

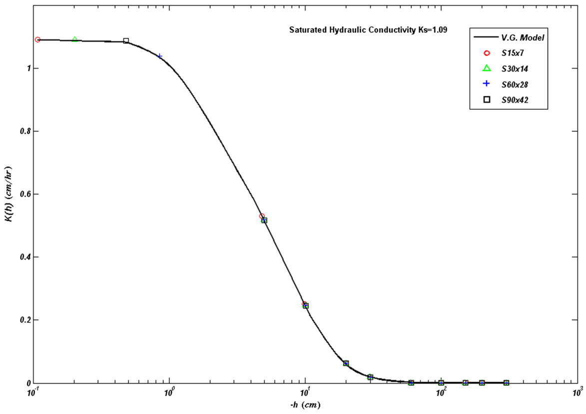

Figure 8. Unsaturated hydraulic conductivity curves of the VG model under different scales (homogeneous media)

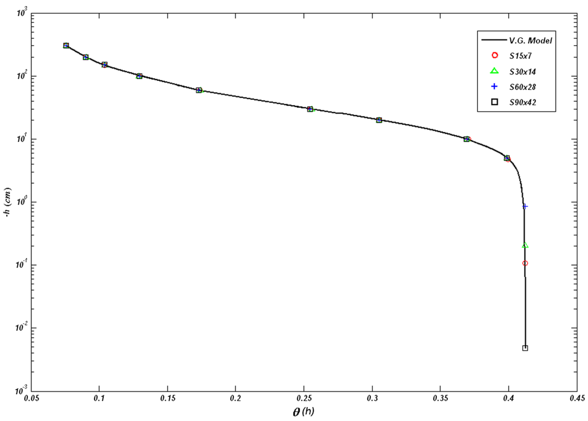

Figure 9. Water retention curves of the VG model under different scales (homogeneous media)

different models of UHC ($\alpha = 0.0028$ and $n = 0.9275$ in Table III) or WRC ($\alpha = 0.3342$ and $n = 1.520$ in Table IV) are no longer the same. This implies that the arrangement of the blocks of contrasting hydraulic properties is crucial to the macroscopic behavior. This has been thoroughly investigated in the article of Wu *et al.* (2011).

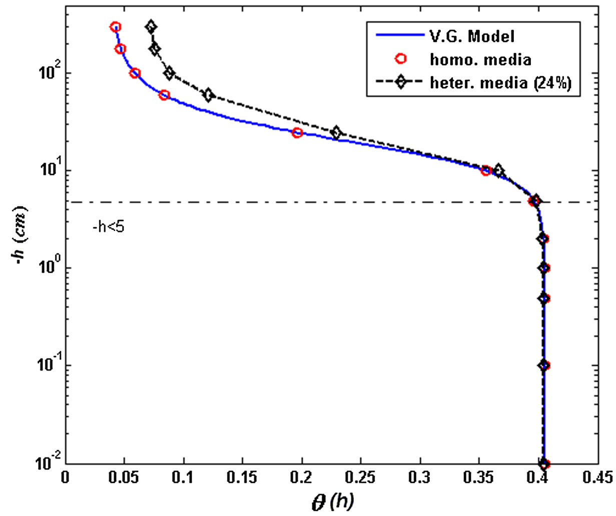

Figure 10. Unsaturated hydraulic conductivity curves of the VG model for homogeneous and heterogeneous media

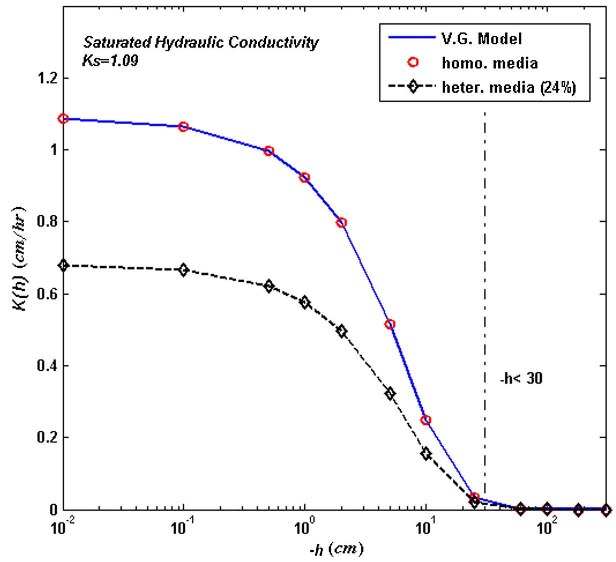

Figure 11. Water retention curves of the VG model for homogeneous and heterogeneous media

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Figure 12 illustrates different scales of soil flumes with local heterogeneity. Four simulation block sizes are the same as the homogeneous ones in Figure 7. Figure 13 shows the variations of WRCs under different scales of heterogeneous media with 24% of heterogeneity. Figure 14 is the variations of UHCs under different scales of heterogeneous media. Figure 15 presents the variations of the $K(h)$-θ relation under different scales of heterogeneous media. Figures 13 and 14 reveal that there is an individual effective curve to represent the overall characteristics of the UHCs and the WRCs, respectively. However, the set of parameters for both UHC and WRC is not substitutive. For instance, the set of parameters obtained from WRC cannot be utilized to predict UHC. From Figures 13 to 15, the results show that the scale affects the parameter estimations for both the WRCs and UHCs. This also suggests that the parameters obtained from the smaller scales are not suitable for adoption by the larger scale simulations. Moreover, the results are consistent with van Genuchten's conclusion that the WRC at low moisture contents is important for an accurate prediction of the UHC; however, the discrepancy of each curve becomes considerable when the moisture content becomes closer to saturation. The reason is that under dry soil conditions, the moisture particles have enough free space to move in porous media and does not necessarily affect the material property (heterogeneity). The soil condition at this specific pressure becomes very dry and results in the relative conductivity to approach zero; therefore, the UHCs overlie each other at the portion representing dry soil conditions. Figure 16 represents variations of α obtained from UHC and WRC under different scales. The solid line with the circular symbol was obtained from UHC, and the dash line with the diamond symbol was obtained from WRC. As can be seen, the discrepancy is significantly large between both curves. The values of α that is estimated from UHC range from 0.0021 to 0.0033, and the ones that are obtained from WRC are approximately 0.23 (Figure 16). There is about a two-order difference of the estimated α between the models under

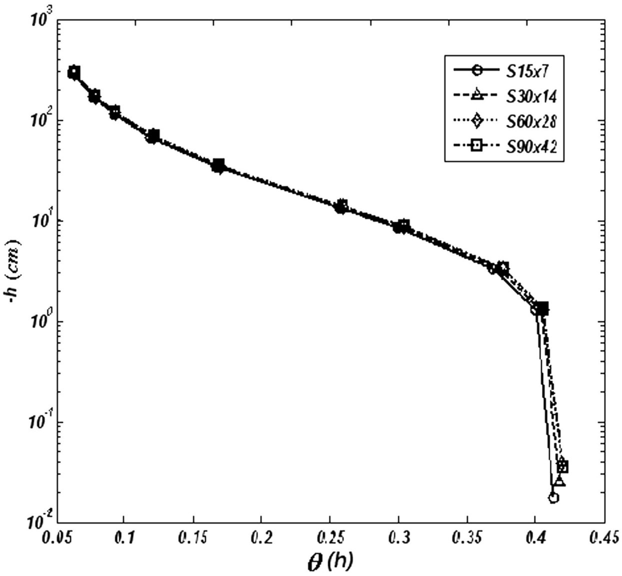

Figure 13. Variations of water retention curves under different scales (VG model)

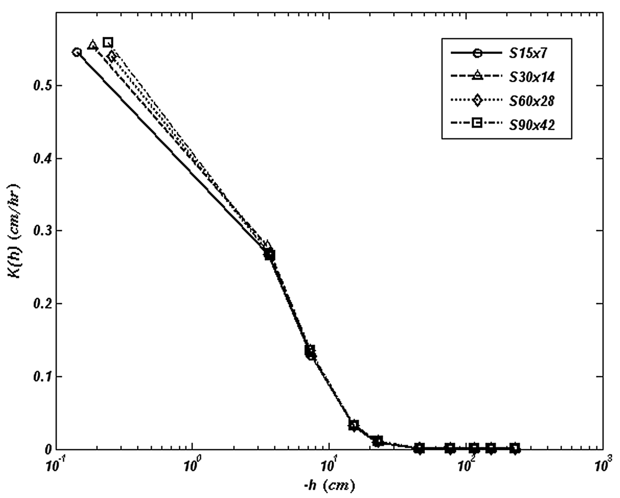

Figure 14. Variations of unsaturated hydraulic conductivity curves under different scales (VG model)

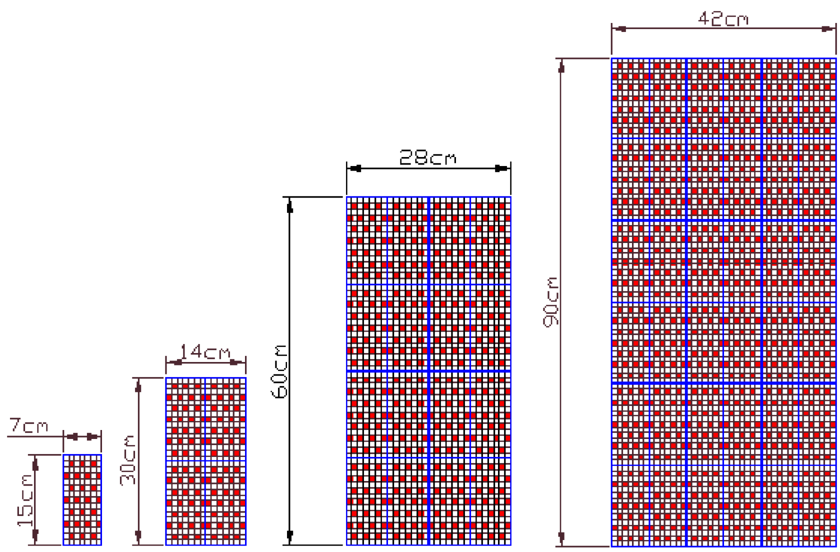

Figure 12. Different scales of soil flumes (heterogeneous media)

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different scales. Notice the values of α that are estimated from WRC are always higher than the ones obtain from UHC. Figure 17 shows variations of n obtained from UHC and WRC under different scales. The values of n that are estimated from UHC range from 0.924 to 0.933, and the ones that are obtained from WRC range approximately from 1.59 to 1.60. The discrepancy is significantly large between both the UHC and WRC curves. Similar to Figure 16, the values of n that are estimated from WRC are always higher than the ones obtained from UHC. The results reveal that the parameters also are affected by scale effects of heterogeneity for both WRC and UHC; this indicates that the parameters obtained from the WRC are not suitable for predicting the UHC at different scales.

Table V shows the fitted parameters of heterogeneous media using UHC and WRC with heterogeneity (24%). Although numerical experiments are implemented under the same conditions, parameters of both α and n that are estimated by UHC reveal a total difference from the ones estimated by WRC. The difference, which is about a two-order discrepancy, is particularly significant for α.

Figures 18–20 are variations of the WRC, UHC, and $K(h)$-θ relation, respectively, under different scales. Regardless of the linear relationship between $K(h)$ and $\theta(h)$, the exponential model has a similar behavior as the VG model. The results reveal that the scale affects the parameter estimations for both the WRC and UHC. Figure 21 represents variations of α that were obtained

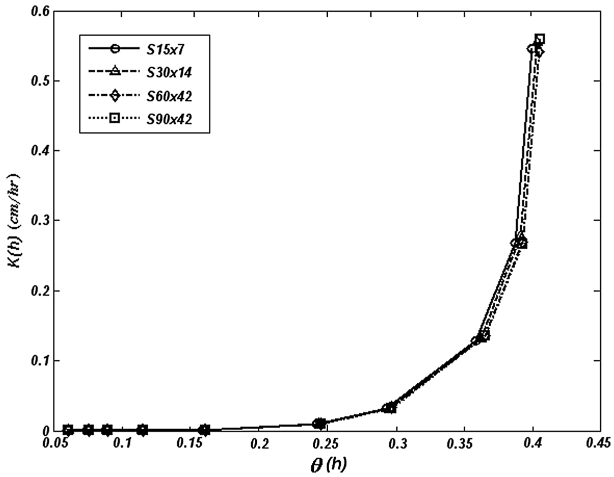

Figure 15. Variations of $K(h)$-θ relation under different scales (VG model)

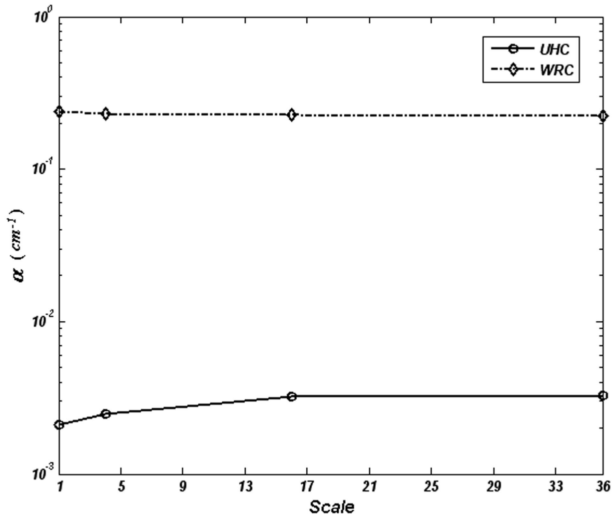

Figure 16. Variations of **α** *versus* scale (fitted by VG model)

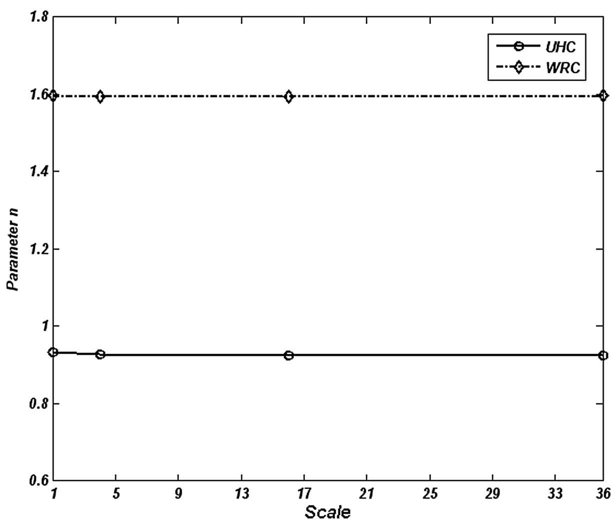

Figure 17. Variations of ***n*** *versus* scale (fitted by VG model)

Table V. Fitted parameters using unsaturated hydraulic conductivity and water retention curve, heterogeneity (24%) [exponential model, Gardner 1958]

Soil flume *cm* x *cm*	Scale	α (cm^{-1}) by UHC	α (cm^{-1}) by WRC	K_s (*cm/h*)
7 × 15	1	0.1715	0.1013	0.763
14 × 30	4	0.1739	0.1044	0.736
28 × 60	16	0.1791	0.1064	0.722
42 × 90	36	0.1747	0.1065	0.715

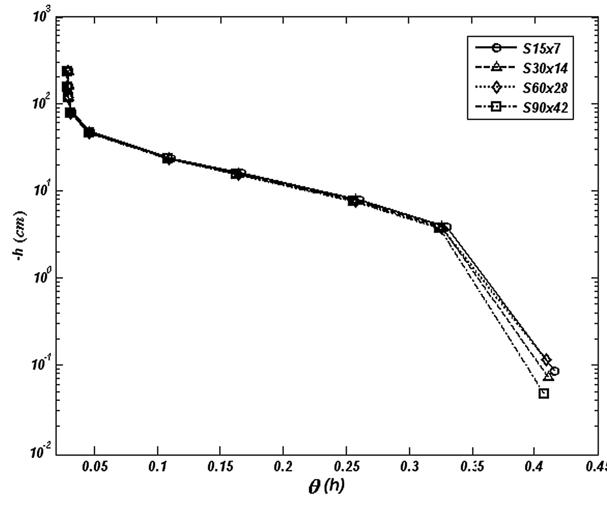

Figure 18. Variations of water retention curves under different scales (exponential model)

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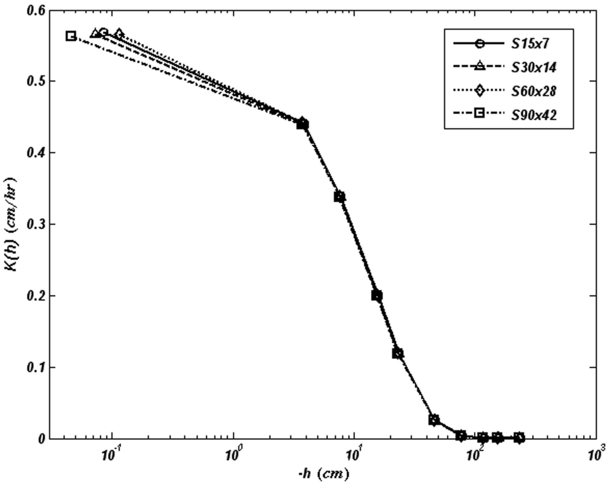

Figure 19. Variations of unsaturated hydraulic conductivity curves under different scales (exponential model)

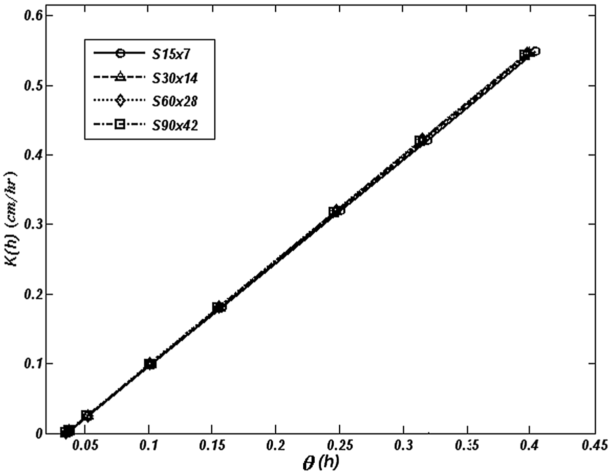

Figure 20. Variations of $K(h)$-θ relation under different scales (exponential model)

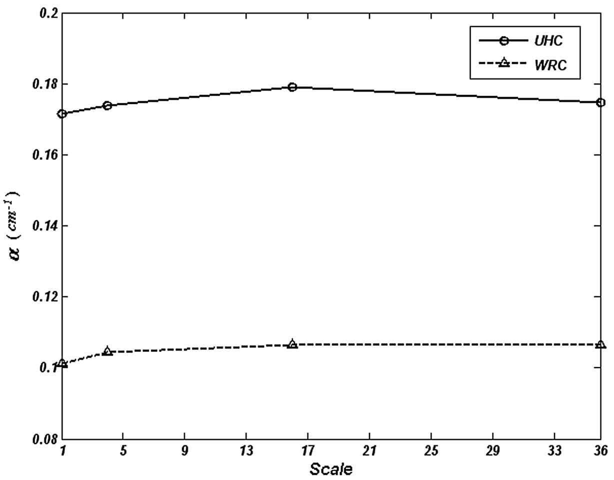

Figure 21. Variations of α *versus* scale (fitted by exponential model)

from UHC and WRC under different scales. The discrepancy of α that was obtained from both the UHC and WRC of the exponential model is not as significant as the one of the VG model. On the contrary to the VG model, the values of α that were estimated from WRC are always lower than the ones obtained from UHC (Table V). This implies that the parameters obtained from the WRC are not suitable for predicting the UHC at different scales.

SUMMARY AND CONCLUSIONS

In homogeneous media, numerical experiments demonstrate that the parameters of soil hydraulic properties are independent of simulation size. Parameters of α and n for every scale in homogeneous media, estimated by using UHC or WRC despite tiny fluctuations, are identical.

Several investigators (Yates *et al.*, 1991; Khaleel *et al.*, 1995) have found that the VG model did not provide accurate estimates of unsaturated hydraulic conductivity for soil sediments with relatively low water contents when these estimates were based solely on retention data and a saturated hydraulic conductivity measurement. For heterogeneous media, the scale affects the results of the parameter estimations when one conducts numerical experiments. Furthermore, the discrepancy of each curve becomes considerable when moisture content becomes closer to saturation. Parameters using UHC are totally different from the ones estimated by WRC. The differences result from the following: (i) local heterogeneity; and (ii) differences of the simulation block size (scale).

Because of the limitation of the numerical model, the magnified experimental soil flumes can be enlarged only to a certain extent. Notwithstanding, the ratio of the simulation size for each case is not large; the scale effect does exist while one conducts a numerical simulation in heterogeneous media. The results reveal that the parameters are affected by the scale effects for both WRC and UHC; this suggests that the parameters obtained from the WRCs are not suitable for predicting the UHCs at different scales for both the VG model and exponential models.

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