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Influence of heterogeneity on unsaturated hydraulic properties (2) – percentage and shape of heterogeneity

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Abstract:

In subsurface porous media, the soil water retention curve (WRC) and unsaturated hydraulic conductivity curve (UHC) are two important soil hydraulic property curves. Spatial heterogeneity is ubiquitous in nature, which may significantly affect soil hydraulic property curves. The main theme of this paper is to investigate how spatial heterogeneities, including their arrangements and amounts in soil flumes, affect soil hydraulic property curves. This paper uses a two-dimensional variably saturated flow and solute transport finite element model to simulate variations of pressure and moisture content in soil flumes under a constant head boundary condition. To investigate the behavior of soil hydraulic property curves owing to variations of heterogeneities and their arrangements as well, cases with different proportions of heterogeneities are carried out. A quantitative evaluation of parameter variations in the van Genuchten model (VG model) resulting from heterogeneity is presented. Results show that the soil hydraulic properties are strongly affected by variations of heterogeneities and their arrangements. If the pressure head remains at a specific value, the soil moisture increases when heterogeneities increase in the soil flumes. On the other hand, the unsaturated hydraulic conductivity decreases when heterogeneities increase in the soil flumes under a constant pressure head. Moreover, results reveal that parameters estimated from both WRC and UHC also are affected by shapes of heterogeneity; this indicates that the parameters obtained from the WRC are not suitable for predicting the UHC of different shapes in heterogeneous media.

KEY WORDS heterogeneity; hydraulic property curve; UHC; WRC; van Genuchten model; VSAFT2

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INTRODUCTION

The soil water retention curve (WRC) and unsaturated hydraulic conductivity curve (UHC) are two important soil hydraulic property curves. The WRC defines the moisture content as a function of the pressure head, and the UHC establishes a relationship between hydraulic conductivities and the pressure head or moisture content. Because field soils are inherently heterogeneous, hydraulic properties of soil have been shown to be variable in both planes and depths, and variations of properties also are spatially correlated (Russo and Bresler, 1981; Vieira *et al.*, 1981; Byers and Stephens, 1983; Greenholtz *et al.*, 1988). In unsaturated flow hydrological modeling, closed-form functions play an important role indirectly in quantifying unsaturated hydraulic data using soil properties that can already or easily be determined (Marison *et al.*, 1994). The UHC can vary in time and space because of its dependence on pressure and moisture content (Yeh *et al.*, 1993, 2005). On the other hand, if the medium is heterogeneous, the parameters (e.g. K_s, α, m, n, θ_s and θ_r) in the WRC and UHC are functions of spatial coordinates.

Parameters of soil hydraulic property can be estimated using empirical models of $\theta(h)$ and $K(h)$ by applying nonlinear curve fitting with moisture content data or unsaturated soil hydraulic data (van Genuchten and Nielsen, 1985; van Genuchten *et al.*, 1991; Marison *et al.*, 1994). In as much as the measurement of unsaturated hydraulic conductivity is considerably more difficult and less accurate than that of the water retention curve, parameters are usually, in actuality, estimated by the water retention curve and then inserted into the unsaturated hydraulic conductivity curve to determine the value of $K(h)$ at a specific pressure (Yeh and Harvey, 1990).

Yeh and Harvey (1990) studied effective unsaturated hydraulic conductivity of layered sands. They concluded that the parameters obtained from water retention curves are not suitable for predicting unsaturated hydraulic conductivity. Another approach for smaller spatial scale heterogeneity was suggested by using the stochastic method (Yeh *et al.*, 1985a,b,c; Hopmans and Stricker, 1989; Yeh, 1989). Leij *et al.* (1997) fitted 14 retention and 11 conductivity functions to 903 sets of data on measurements of soil and rock samples to evaluate a

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variety of functional expressions. Zhu and Mohanty (2002) and Zhu *et al.* (2006) used spatial averaging schemes to evaluate effective parameters for a steady-state flow in heterogeneous soils.

To determine the influence of stones on hydraulic conductivity of saturated soils, Sauer and Logsdon (2002) identified, using infiltration tests, a small increase of effective saturated hydraulic conductivity (K_{se}) with an increase of the stone content in two soil types at the pressure head of 12 cm. They estimated the relationship of K_{se} with the ratio of the volumetric stone and the soil volume using infiltration tests, and their results were most likely explained by spatial variability of both variables. Novák *et al.* (2011) used a two-dimensional simulation model to show the development of the unsaturated zones located below the stones, which depended on their sizes and shapes. *Chen et al.* (2011) used numerical experiments to examine influences of local heterogeneities and scale effects on soil hydraulic properties of the VG model (van Genuchten, 1980) in subsurface porous media. Their results showed that soil hydraulic properties are strongly affected by variations of local heterogeneities, and simulation scale affects the results of the parameter estimations when numerical experiments were carried out. The discrepancies of UHC and WRC became considerable when the moisture content became closer to saturation. Parameters using UHC were totally different than the ones estimated by WRC.

Disregarding the variety of approaches, the fundamental question is as follows: how can this heterogeneity be characterized and incorporated into a quantitative description of flow and solute transport in an unsaturated system?

Although closed-form functions for unsaturated hydraulic properties are widely employed, it appears that relatively little literature exists on examining the influence of heterogeneity on parameters of models. Therefore, the purposes of the present study are two-fold: (i) to evaluate variations of parameters for soil hydraulic properties of the VG model in different amounts of heterogeneity; and (ii) to investigate the influences of the pattern of heterogeneity on the parameters of the VG model.

Using numerical experiments, the different amounts of heterogeneity and heterogeneous shapes were demonstrated. A quantitative evaluation of parameter variations in the VG model resulting from heterogeneity was investigated.

A CONTROVERSY OF THE VG MODEL USED IN HETEROGENEOUS MEDIA

Simulation of unsaturated flow and solute transport typically uses closed-form functions of the soil hydraulic property model, namely, the soil WRC and UHC. The WRC defines the water content as a function of the water pressure head, and the UHC establishes a relationship between hydraulic conductivities and water pressure head or water content. These closed-form functions facilitate rapid estimations of the soil hydraulic properties of different media for flow and solute transport modeling.

The VG model (van Genuchten, 1980) has become one of the most widely used curves for characterizing soil hydraulic properties. van Genuchten (1980) identified an S-shaped function that fits very well to measured water-retention characteristics of many types of soil. It is expressed as

$$\theta(h) = (\theta_s - \theta_r)[1 + (\alpha|h|)^n]^{-m} + \theta_r \qquad (1)$$

where h is the capillary pressure head, θ_s is the saturated moisture content, and θ_r is the residual moisture content; α $[1/L]$ is the pore-size distribution parameter, representing the rate of reduction in conductivity as the soil desaturates; n [ ] and m [ ] are soil parameters and $m = 1 - 1/n$. Equation (1) characterizes laboratory-measured water retention well over the typical range of suction (Russo, 1988; Hill *et al.*, 1989; Michiels *et al.*, 1989; Vereeken *et al.*, 1989; Yeh *et al.*, 1993, 2005), and many studies have reported water retention data in terms of the VG model (Carsel and Parrish, 1988; Russo, 1988; Nimmo, 1991). Equation (1) with $m = 1$ has been successfully applied in many studies to describe soil–water retention data (Ahuja and Swartzendruber, 1972; Haverkamp *et al.*, 1977). This function was combined with Mualem's hydraulic conductivity function (Mualem, 1976a) to predict unsaturated hydraulic conductivity.

$$K(h) = K_s \frac{\left(1 - (\alpha h)^{n-1}[1 + (\alpha|h|)^n]^{-m}\right)^2}{[1 + (\alpha|h|)^n]^{m/2}} \qquad (2)$$

where K_s is the saturated hydraulic conductivity.

The controversial issues of applying Equations (1) and (2) to heterogeneous media are as follows:

I. Equation (1) is where measured water-retention characteristics of real soils are fitted to an empirical model (i.e. Hygiene sandstone, Touchet silt loam, silt loam, etc.).
II. Equation (2) is based on Maulem's theoretical pore-size model (Mualem, 1976b), which considers inter-connected pores of a homogeneous porous medium.

Because Equation (1) is based on specific soil samples being fitted to an empirical function, and Maulem's model is derived under the assumption of a homogeneous porous medium, there is an irrational linkage between Equations (1) and (2) because the two equations are derived from opposite media: Equation (1) involves heterogeneity, and Equation (2) involves homogeneity. Notwithstanding the conclusions that van Genuchten (1980) made ('a reasonable description of the WRC at low water contents was important for an accurate prediction of the UHC by comparing five experimental data'), application of the VG model by using parameters of WRC to predict UHC in heterogeneous media is still questionable. This will be presented in the following section.

For practical application, the parameters of Equation (2) are usually estimated by the WRC from the measurements of h and θ, and then, they are placed into the unsaturated hydraulic conductivity curve to predict the value of $K(h)$ at a specific pressure. The VG model

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is valid over a broader range of pressure values than the exponential model (van Genuchten and Nielsen, 1985), and the parameters α and n are identical in homogeneous media. Moreover, it closely fits the measured water-retention data of many types of unsaturated soils (Leij *et al.*, 1997).

By transferring Equation (2) into another form (Brooks and Corey, 1964), we obtain

$$S_e = \frac{\theta(h) - \theta_r}{\theta_s - \theta_r} = \left(\frac{h}{h_d}\right)^{-\lambda}, \quad \text{when } \frac{h}{h_d} > 1 \qquad (3)$$

where S_e is the effective saturation; $h_d = \frac{1}{\alpha}$; $\lambda = -nm$ is a parameter that defines the relationship between the moisture content and negative pressure head h affecting the slope of the WRC.

Because of the use of the above mathematical models for the functional relation between the unsaturated hydraulic conductivity, pressure head and moisture content, soils often can be categorized by parameters such as α, n, θ_s, θ_r and Ks. For example, coarse-textured soils are reported to have large values of α, n and Ks, and fine-textured soils are reported to have small values (Stephens and Rehfeldt, 1985). However, values of these parameters are not necessarily unique for a given geological medium because of the hysteretic behavior between the $K(h)$ and $\theta(h)$ relationship; these values can be different according to the wetting and drying histories of the medium. Furthermore, parameters for both WRC and UHC of the VG model are identical for homogenous porous media without a doubt, but it is still uncertain if this identical relationship remains alike in a heterogeneous one.

PARAMETER OPTIMIZATION

In practice, WRC of soil can be measured at less cost and take less effort than that of unsaturated hydraulic conductivity. Parameters of α and n of UHCs and WRCs often are conveniently assumed to be the same, although they may be different (e.g. Yeh and Harvey, 1990).

In this paper, the observed data of water content *versus* pressure head and the unsaturated hydraulic conductivity *versus* pressure head were obtained through numerical experiments. Accordingly, the parameters of the VG model were estimated either from the UHC or WRC. The parameter values of both α and n were determined by minimizing the objective function,

$$\min f_\theta = \sum_{i=1}^{N_1} \left[\theta_i(h) - \hat{\theta}_i(h)\right]^2 \qquad (4)$$

where h is the pressure head; θ_i and $\hat{\theta}_i$ are the observed and fitted moisture contents, respectively; N_1 is the number of observed moisture contents, or else, it is obtained by minimizing the objective function,

$$\min f_K = \sum_{i=1}^{N_2} \left[K_i(h) - \hat{K}_i(h)\right]^2 \qquad (5)$$

where $K_i(h)$ and $\hat{K}_i(h)$ are the observed and fitted unsaturated hydraulic conductivities, respectively; N_2 is the number of observed unsaturated hydraulic conductivities. The optimization was carried out according to the Levenberg–Marquardt method (Marquardt, 1963). Equations (4) and (5) were applied to the WRC and UHC, respectively, to estimate the parameters of α and n.

NUMERICAL EXPERIMENTS

The numerical experiments were conducted to examine the behavior of the soil hydraulic property curve owing to variations of heterogeneities and their arrangements as well. The VSAFT2 (Yeh *et al.*, 1993) was used to implement numerical experiments with specific soil parameters under different media conditions (i.e. heterogeneities). A synthetic soil flume with dimensions of 28×60 cm was established for the numerical experiments, and heterogeneity was set up accordingly.

Experimental setup

To examine the effect of local heterogeneity on soil hydraulic properties through the unsaturated zone, synthetic media flumes were simulated using VSAFT2 to simulate variations of pressure and moisture content in the soil flume under a constant head boundary condition. Under given boundary conditions, the moisture content and specific flux were obtained at each grid point, and then, the mean moisture content under specific pressure heads of the larger simulation sizes were calculated. The unsaturated hydraulic conductivity was obtained for the different pressure heads by the soil hydraulic property model. A steady-state flow simulation was carried out on unsaturated heterogeneous porous media. Two types of soil were used for numerical experiments, and their parameters are listed in Table I. The experimental flume

Table I. Parameters of soil for the van Genuchten model

Parameters		θ_s	θ_r	K_s (cm·h^{-1})	α (cm^{-1})	n
Soil type						
1×1 cm*	Sandy loam	0.412	0.041	1.09	0.0523	1.857
	Clay	0.385	0.180	0.03	0.0270	1.600

*Grid size for numerical experiment in Figure 3.

contained predominantly sandy loam soil embedded with clay to create heterogeneity. The UHC of the VG model for both sandy loam soil and clay soil is shown in Figure 1, and the WRC of the VG model for both sandy loam soil and clay soil is in Figure 2. The dimensions of the experimental soil flume were 28×60 cm, and the dimensions of each numerical discrete cell were 1×1 cm as shown in Figure 3(a). The numerical model was simulated with the specific boundary conditions at the top and the bottom of the flume as given in Table II. As can be seen, the same pressure heads are given for the top boundary and bottom boundary. The total head of the upper boundary equals to the head of the lower boundary plus the height of the experimental flume. The experiments were carried out by two categories on the unsaturated hydraulic conductivity curve and water retention curve: (i) the influence of different percentages of heterogeneity; and (ii) the influence of heterogeneous shapes.

Heterogeneity setup

To examine the effects of different amounts of heterogeneity on the parameters of the VG model, four different actual soil flumes were set up, and experiments were conducted numerically. The first contained homogeneous sandy loam with $\theta_s = 0.412$, $\theta_r = 0.041$, $\alpha = 0.0523\,\mathrm{cm}^{-1}$, $K_s = 1.09\,\mathrm{cm{\cdot}h}^{-1}$, and $n = 1.857$ for each cell grid of 1×1 cm. The second contained homogeneous sandy loam embedded with 6% of local heterogeneity (clay) with $\theta_s = 0.385$, $\theta_r = 0.18$, $\alpha = 0.027\,\mathrm{cm}^{-1}$, $K_s = 0.03$ $\mathrm{cm{\cdot}h}^{-1}$, and $n = 1.6$ for each cell dimensions of 1×1 cm. Clay blocks with a constant size of 1×1 cm were embedded as the local heterogeneity. The third and the fourth contained homogeneous sandy loam embedded with 12% and 24% local heterogeneity, respectively. In Figure 3, flumes (a) to (d) illustrate the configurations of the experimental soil flumes with different amounts of heterogeneity: (a) homogeneous media (0%), (b) 6% heterogeneity, (c) 12% heterogeneity, and (d) 24% heterogeneity.

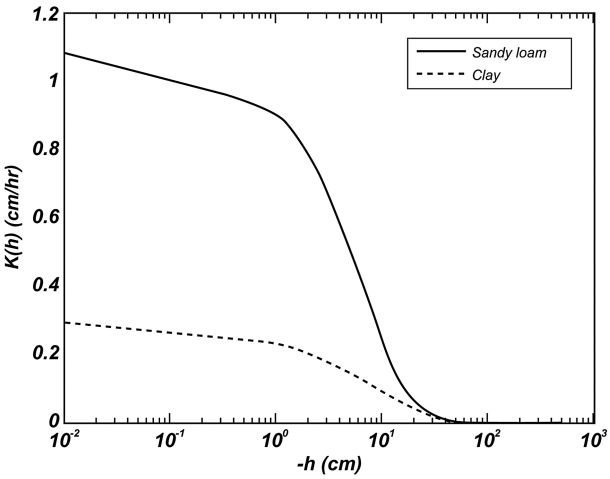

Figure 1. Unsaturated hydraulic conductivity curve (UHC) of the van Genuchten model (VG model) for both sandy loam soil and clay soil

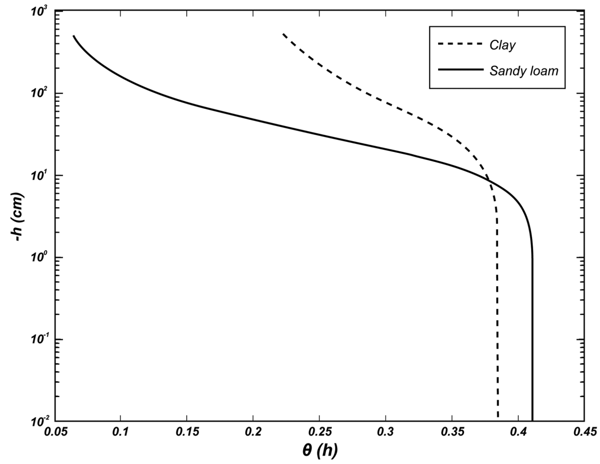

Figure 2. Water retention curve (WRC) of the VG model for both sandy loam soil and clay soil

On the other hand, there was an interest in how the heterogeneous shapes affect the parameters of the VG model if the heterogeneity remains at the same amount but instead with different heterogeneous shapes. To examine this situation, case (c) (with 12% heterogeneity) was rearranged into five different shapes. In Figure 4, shapes 1–5 show different shapes of heterogeneity in the soil flumes. Although these shapes are totally assumed (undoubtedly, the natural situation is much more complicated!), it is helpful to understand how heterogeneous shapes affect the parameters of the VG model.

Values of parameters α and n for both UHC and WRC for each pattern were obtained by conducting numerical experiments using VSAFT2, and then, the nonlinear least squares minimization of Equations (4) and (5) was applied to determine both α and n values based on the same tolerance of error.

ANALYSIS OF RESULTS

Two heterogeneous situations were simulated to examine the influence of heterogeneity: (A) effects of different amounts of heterogeneity on parameters of the VG model; and (B) effects of different heterogeneous shapes on parameters of the VG model. Corresponding results of numerical experiments for both (A) and (B) are discussed in the following section.

Effects of different amounts of heterogeneity on parameters of the VG model

The parameters of hypothetical sandy loam soil are given in Table I. A non-linear least-square curve fitting was used to find the best-fit values of α and n in both Equations (1) and (2) of the VG model, namely, WRC and UHC.

Figure 5 presents the fitted parameters of α (0.0523) and n (1.857) using UHC for a homogeneous soil flume. The circular symbols are results of numerical experiments, and the solid line is a fitted curve of theoretical UHC of the VG model. Similarly, Figure 6 shows the

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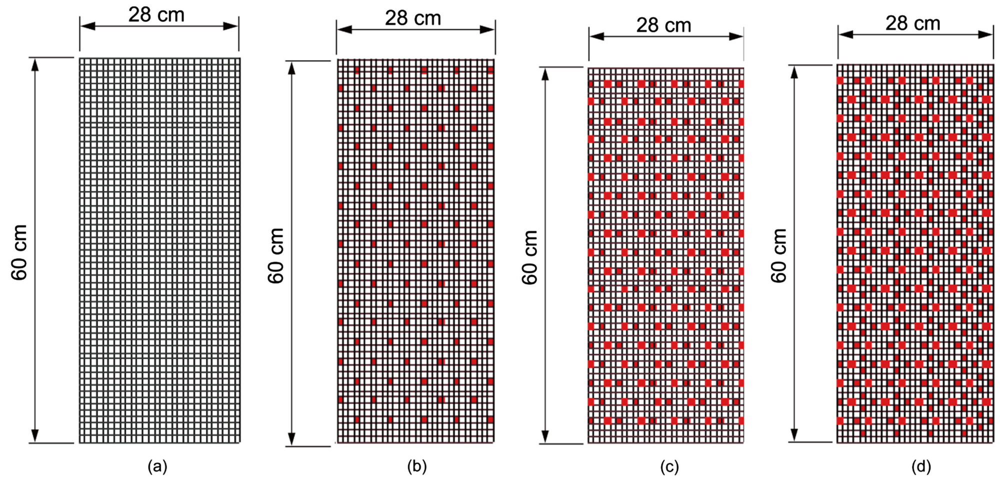

Figure 3. Configurations of experimental soil flumes: (a) homogeneous media (0%), (b) 6% heterogeneity, (c) 12% heterogeneity and (d) 24% heterogeneity

Table II. Boundary conditions for numerical experiment

Exp. no.	1	2	3	4	5	6	7	8	9	10	11	12
Upper B.* (cm)	−0.00001	−1	−5	−10	−20	−30	−40	−50	−100	−150	−200	−300
Lower B.* (cm)	−0.00001	−1	−5	−10	−20	−30	−40	−50	−100	−150	−200	−300

*Pressure head (cm).

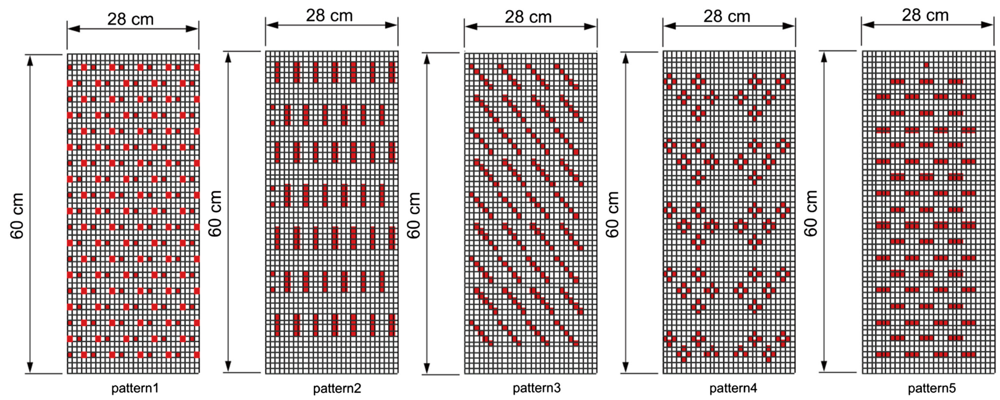

Figure 4. Shapes of heterogeneity in soil flumes: 12% heterogeneity

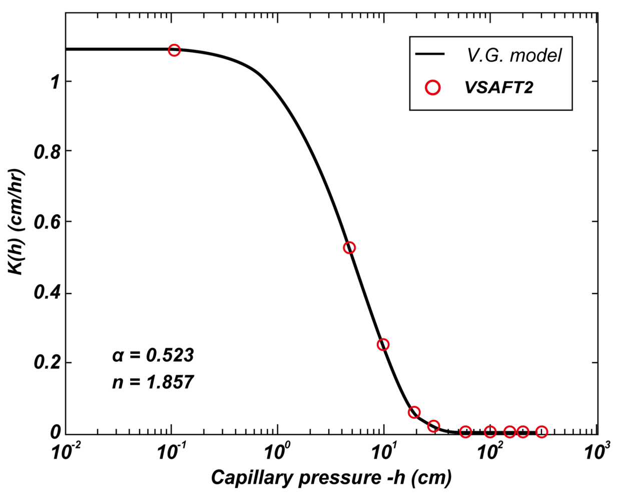

Figure 5. Unsaturated hydraulic conductivity curve: homogeneous soil flume

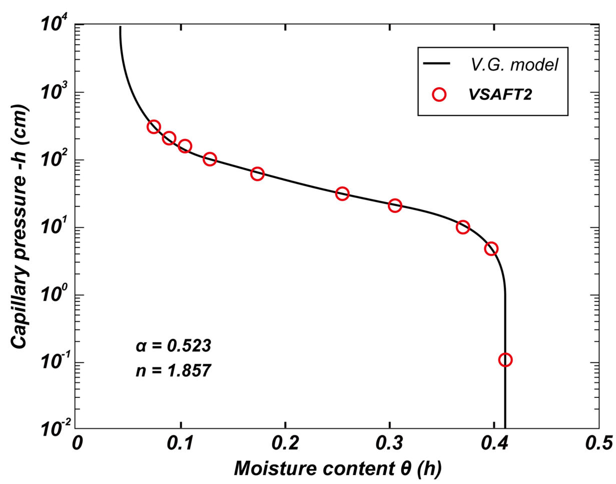

Figure 6. WRC: homogeneous soil flume

fitted parameters of α (0.0523) and n (1.857) of the same soil flume using WRC. As can be seen, both α and n are identical for homogeneous media regardless of using UHC or WRC. These results strongly support that parameters α and n are the same in a homogeneous case for both WRC and UHC from the original theoretical derivation, and numerical experiments of VSAFT2 show the same results as well.

Using numerical experiments to simulate the heterogeneous media, the parameter set of α and n obtained from the different models of UHC and WRC, respectively, are no longer the same. Figure 7 represents variations of unsaturated hydraulic conductivity curves with respect to different amounts of heterogeneity. Under a high moisture content with a specific pressure h, the values of unsaturated hydraulic conductivity decrease when the amounts of heterogeneity increase. As can be seen, when the negative pressure head is higher than a certain value (i.e. $h > -30$ cm), all UHCs together overlie each other. This indicates that the soil condition at this specific pressure becomes very dry and results in the relative conductivity to approach zero. On the contrary, when the negative pressure head is low, the soil moisture increases, and the discrepancy of each UHC under different percentages of heterogeneity increases.

Figure 8 shows variations of the water retention curve with respect to different percentages of heterogeneity. When the pressure head remains fixed, the values of the water content increase when the amounts of heterogeneity increase. As can be seen, when the negative pressure head is less than a certain value (i.e. $h < -5$ cm), each WRC remains constant. This behavior follows the intuitive physical sense of a pressure–moisture relationship in unsaturated porous media. The vertically straight part of WRC in Figure 8 is equivalent to the S_e of the Brooks and Corey model (Brooks and Corey, 1964). Well known is that the height of S_e depends on the percentages of heterogeneity as well. This indicates that the soil condition at this specific pressure becomes

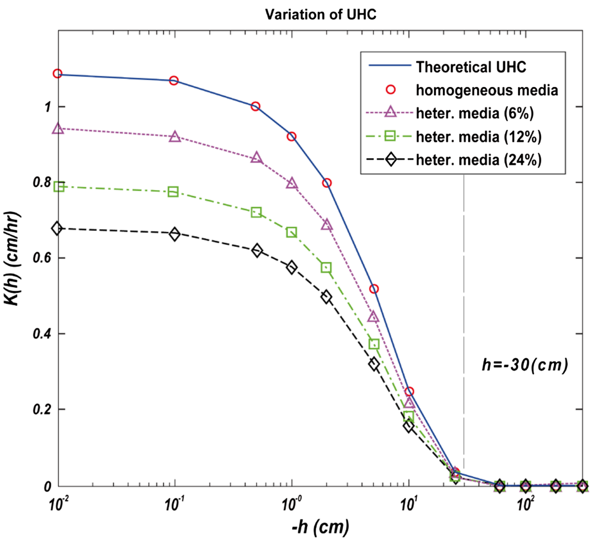

Figure 7. Variations of UHC with respect to different percentages of heterogeneity

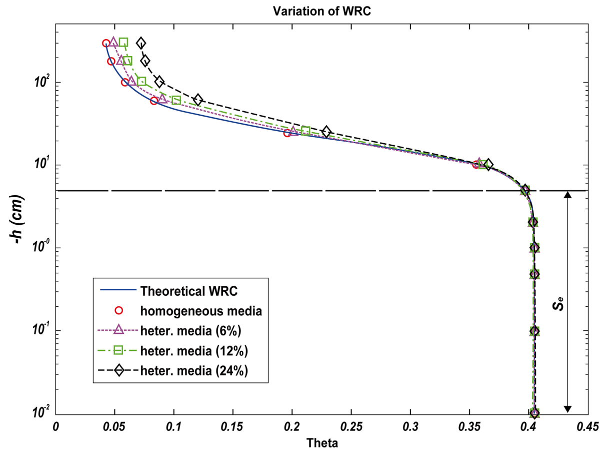

Figure 8. Variations of WRC with respect to different percentages of heterogeneity

saturated, and the moisture content remains constant. Figure 8 also reveals that all WRCs are close to each other and are parallel when the soil is dry. In both Figures 7 and 8, when the soil condition is dry, the results are consistent with van Genuchten's (1980) conclusion that the WRC at low moisture contents is important for an accurate prediction of the UHC. The reason is that under dry soil, moisture particles have enough free space to move in porous media. Material property (e.g. heterogeneity) does not necessarily affect the moving of moisture particles. The soil condition at this specific pressure becomes very dry and results in the relative hydraulic conductivity to approach zero; therefore, the UHCs overlie each other at the portion representing dry soil. In addition, we would like to point out why the WRCs of different amounts of heterogeneity in Figure 8 do not intersect with each other like the ones in Figure 2. The plotted result of the WRCs in Figure 8 has been subtracted from the heterogeneity portion of the soil flume. Therefore, the results of the WRCs in Figure 8 reflect the net WRCs of sandy loam soil.

Figures 9 and 10 are the fitted parameters of α and n, respectively. Figure 9 shows the fitted values of α using

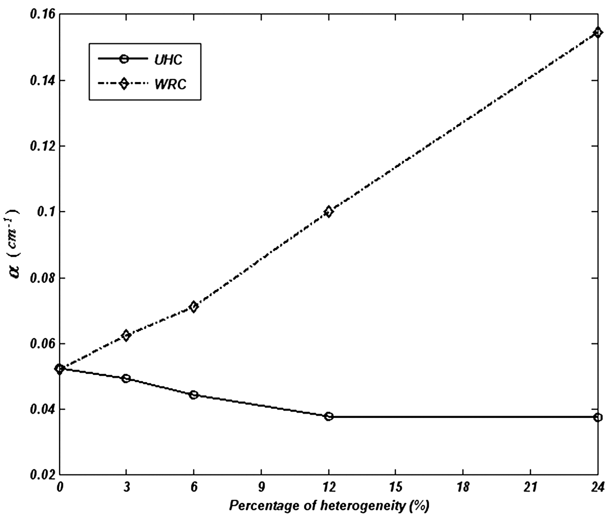

Figure 9. Parameter α obtained from UHC and WRC

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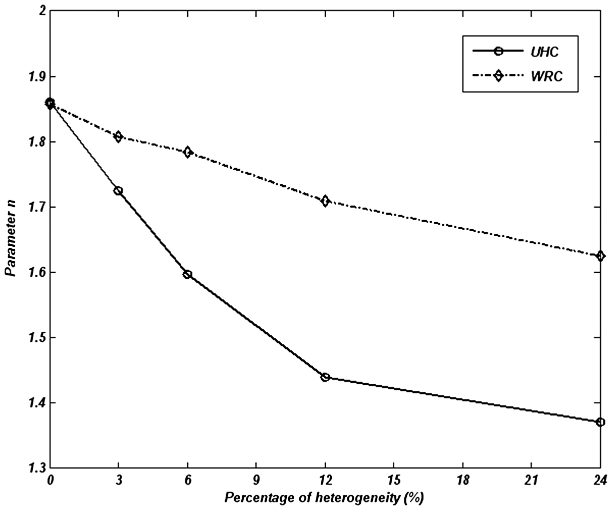

Figure 10. Parameters of n obtained from UHC and WRC

UHC (the solid line with circular symbols) and WRC (the dash line with diamond symbols) obtained from Figures 7 and 8, respectively. As can be seen, the value of α obtained from UHC coincides with the one obtained from WRC for homogeneous media (0%). This confirms the assumption that the parameters of α and n of unsaturated hydraulic conductivity and water retention curves are the same in homogeneous porous media. However, the discrepancy becomes significant when the amounts of local heterogeneity increase. Notice that the trend of α obtained from UHC is totally different than that of WRC. The value of α obtained from WRC increases when the amounts of local heterogeneity increase, but it represents a reversed behavior than that obtained from UHC.

Figure 10 shows values of fitted n using UHC and WRC. Similar to Figure 9, the solid line with circular symbols is the fitted values using UHC, and the dash line with diamond symbols is the fitted values using WRC obtained from Figures 7 and 8, respectively. The value of n obtained from UHC coincides with the one obtained from WRC for homogeneous media (0%). Although the value of n shows the same trend (both decrease), the discrepancy is significant as the amounts of local heterogeneity increase. Table III lists the fitted parameters α and n using the UHC. Values of α range between 0.0525 and 0.0376, and values of n range between 1.370 and 1.860. Table IV is the fitted parameters α and n using the WRC. Values of α range between 0.0523 and 0.1544, and values of n range between 1.624 and 1.857. Figure 11 shows the variations of K_s owing to different amounts of heterogeneity. It reveals that saturated hydraulic conductivity decreases when the amounts of local heterogeneity increase.

Effects of different heterogeneous shapes on parameters of the VG model

Figure 4 shows configurations of shapes 1–5 that are allocated in different arrangements with the same percentages (12%) of heterogeneity. From the figure, it can be seen that pattern 2 forms a horizontal stratification, and pattern 5 forms a vertical stratification. Figures 12 and 13 show the variations of UHC and WRC with respect to different shapes. As can be seen from both Figures 12 and 13, the heterogeneous shapes affect the UHC and WRC. The discrepancy of UHC becomes obvious when the soil is close to saturation. On the contrary, the WRC is totally different when the soil is close to dry conditions. In addition, Figures 12 and 13 show that there is an individual effective curve to represent the overall characteristics of UHC and WRC, respectively. However, a set of parameters for both UHC and WRC is not substitutive. For instance, the set of parameters obtained from the WRC cannot be utilized to predict the UHC.

Figure 14 represents variations of α obtained from UHC and WRC under different shapes of heterogeneity. The solid line with circular symbols was obtained from UHC, and the dash line with diamond symbols was obtained from WRC. As can be seen, the discrepancy is significant when obtained from both equations. Values of α estimated from UHC range from 0.0181 to 0.0458

Table III. Fitted parameters using unsaturated hydraulic conductivity curve

Soil flume	Percentage of heterogeneity	α (cm^{-1})	n	K_s (cm·h^{-1})
28 × 60	Homogeneous	0.0525	1.860	1.090
	(3%)	0.0495	1.725	1.033
	(6%)	0.0444	1.597	1.015
	(12%)	0.0380	1.440	0.915
	(24%)	0.0376	1.370	0.830

Table IV. Fitted parameters using water retention curve

Soil flume	Percentage of heterogeneity	α (cm^{-1})	n	K_s (cm·h^{-1})
28 × 60	Homogeneous	0.0523	1.857	1.090
	(3%)	0.0624	1.808	1.033
	(6%)	0.0712	1.784	1.015
	(12%)	0.1001	1.709	0.915
	(24%)	0.1544	1.624	0.830

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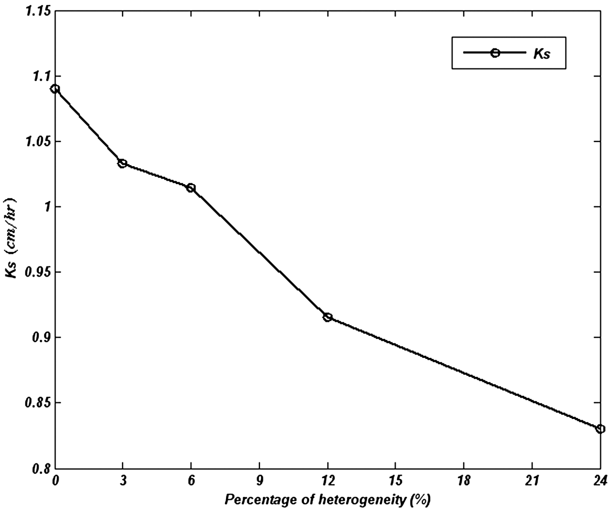

Figure 11. Variations of K_s because of different amounts of heterogeneity

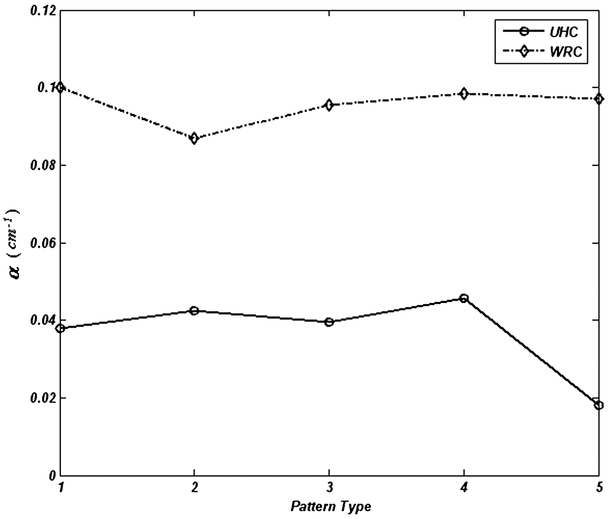

Figure 14. Variations of parameter α *versus* shapes of heterogeneity

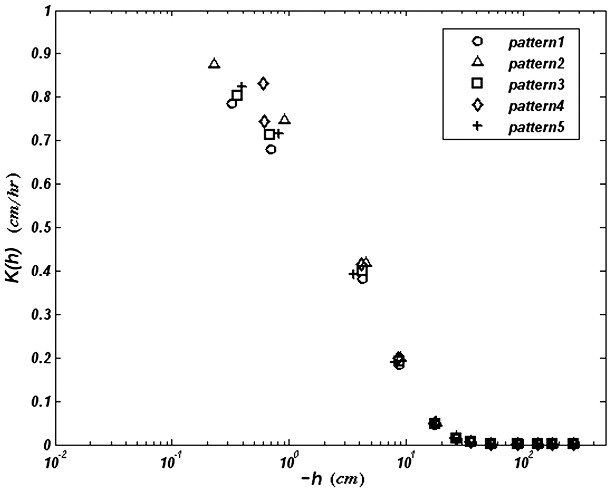

Figure 12. Variations of UHC with respect to different shapes

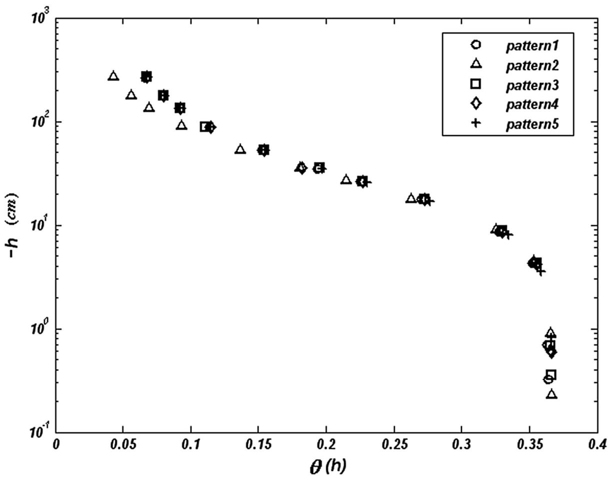

Figure 13. Variations of WRC with respect to different shapes

(Table V), and the ones obtained from WRC range from 0.0870 to 0.1 (Table VI). There is about a one order difference of estimated α between the models with different shapes of heterogeneity. Notice that the values of α estimated from WRC are always higher than the ones obtained from UHC.

Figure 15 shows variations of n obtained from UHC and WRC with different shapes of heterogeneity. The discrepancy is significant when obtained from both UHC and WRC. Similar to Figure 14, values of n estimated from WRC are always higher than the ones obtained from UHC. Figure 15 reveals that parameters α and n estimated from UHC result in the highest value for pattern 2 (parallel to flow direction) and result in the lowest value for pattern 5 (perpendicular to flow direction).

Table V lists the fitted parameters using the UHC, and Table VI represents the fitted parameters using the WRC. Figure 16 shows the variations of K_s *versus* shapes of heterogeneity. The saturated hydraulic conductivity fluctuates when the shapes of local heterogeneity change. It reveals that pattern 2 results in the highest value for heterogeneity, being parallel to flow direction, and pattern 5 results in the lowest value for heterogeneity, being perpendicular to flow direction.

The results reveal that the parameters also are affected by shapes of heterogeneity for both WRC and UHC; this indicates that the parameters obtained from the WRC are not suitable for predicting the UHC of different shapes in heterogeneous media.

SUMMARY AND CONCLUSIONS

In this study, we examined variations of parameters for the soil hydraulic property of the VG model in different amounts of heterogeneity as well as influences of the shapes of heterogeneity on the parameters of the VG model. From numerical experiments and their analyses, results indicate that the parameters estimated from the WRC are totally different from the ones estimated from the UHC in heterogeneous media. Parameter α obtained from WRC increases when the amounts of local heterogeneity increase, but it represents a reversed behavior when obtained from UHC. Parameter n shows the same trend (both decrease), but the discrepancy is

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Table V. Fitted parameters using unsaturated hydraulic conductivity curve

Soil flume	Pattern of heterogeneity	α (cm^{-1})	n	K_s (cm·h^{-1})
28×60 cm	1	0.0380	1.440	0.915
	2	0.0458	1.538	0.907
	3	0.0424	1.531	0.938
	4	0.0395	1.470	0.907
	5	0.0181	1.294	0.747

Table VI. Fitted parameters using water retention curve

Soil flume	Pattern of heterogeneity	α (cm^{-1})	n	K_s (cm·h^{-1})
28×60 cm	1	0.1001	1.709	0.915
	2	0.0983	1.718	0.907
	3	0.0870	1.895	0.938
	4	0.0956	1.724	0.907
	5	0.0970	1.713	0.747

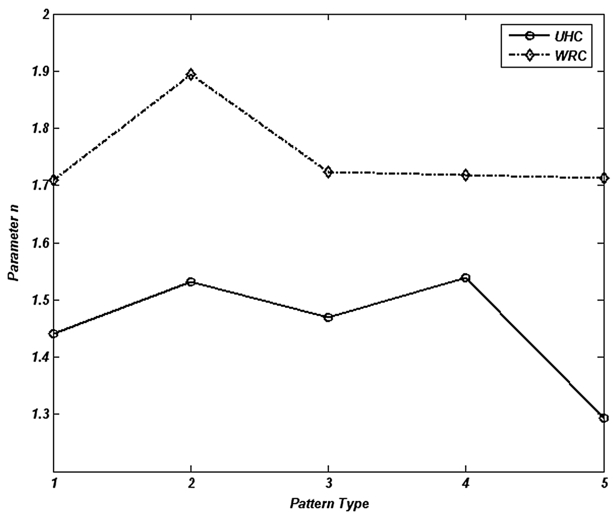

Figure 15. Variations of parameter n *versus* shapes of heterogeneity

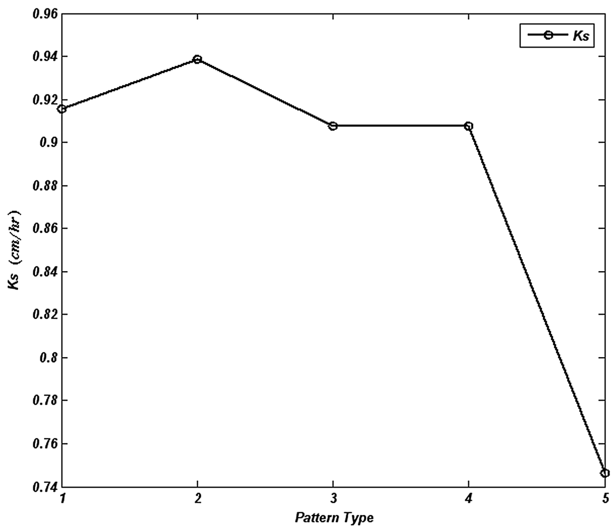

Figure 16. Variations of K_s *versus* shapes of heterogeneity

significant as the amounts of local heterogeneity increase. As a result, both α and n obtained from WRC are not suitable for predicting the UHC in heterogeneous media, which is consistent to the conclusion made by Yeh and Harvey (1990).

Furthermore, the estimated parameters are affected by different arrangements with the same amounts (12%) of heterogeneity for both WRC and UHC. Values of both α and n estimated from WRC are always higher than the ones obtained from UHC.

Finally, in this study, we raised several points of interest concerning the variations of parameters for the VG model affected by percentages of heterogeneity and its pattern under the same amount of heterogeneity. Numerical experiments illustrate that the VG model is applicable for homogeneous media with the same parameters obtained from either UHC or WRC. However, in heterogeneous soil, both α and n are different for UHC and WRC, respectively; they have to be calibrated for individual models or be combined using a composite objective function of the parameter optimization (Leij *et al*., 1997).

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