Time and Flow Characteristics of Component Hydrographs Related to Rainfall–Streamflow Observations

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Abstract: This study investigates the shape characteristics of hydrograph components of the Wu-Tu watershed in Taiwan based on observations of rainfall and streamflow. Component hydrographs were modeled using a model of three serial tanks with one parallel tank. The block kriging method was used to calculate the hourly mean rainfall of events, and eight model parameters of 34 cases were derived from the shuffled complex evolution optimal algorithm. The remaining 18 events were used to verify the applicability of the calibrated parameters. Results show that (1) times to peak of hydrograph components are positively nonlinearly correlated to peak time of rainfall; (2) peak discharges of hydrograph components are linearly proportional to those of streamflow hydrograph; and (3) relationships of total discharges also have direct ratios between hydrograph components and observed streamflow. Using the procedures proposed in this study, three evaluated shape characteristics of component hydrographs can be easily used to rapidly determine shapes of simple hydrographs. **DOI: 10.1061/** (ASCE)HE.1943-5584.0000675. © 2013 American Society of Civil Engineers.

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Introduction

Many hydrologists have developed rainfall-runoff conceptual models. Approximations of the convolution integral are commonly used to derive conceptual rainfall-runoff models and generate outlet runoffs of a watershed. Models derived from the convolution integral are generally known as unit hydrograph (UH)-based models. Derivations with specific parameters include the Nash model (Nash 1957; Cheng and Wang 2002; Cheng et al. 2008b, 2010; Huang et al. 2008a, b), mathematical models (Clarke 1973; Ahmad et al. 2009), geomorphologic instantaneous unit hydrograph (GIUH) models (Jin 1992; Franchini and O'Connell 1996; Nourani et al. 2009), the distributed parallel model (Hsieh and Wang 1999), and subwatershed divisions (Agirre et al. 2005); rainfallrunoff processes (O'Connell and Todini 1996; Melone et al. 1998; Bhadra et al. 2010) have been modeled with IUH. Other models that differ from the UH-based model are the Muskingum-Cunge modeling (Ponce and Lugo 2001) and the nonlinear kinematic wave model (Mizumura and Ito 2011).

Archived rainfall and streamflow data is essential to the development of UH-based models (e.g., the Nash model). Applying these UH-based models involves determining both effective rainfall and direct runoff of a rainfall-runoff event in advance. The direct runoff, which is computed separately from the base flow, is a streamflow component. Base flow is frequently considered to be a constant in a rainfall-runoff event. Previous studies addressed the effects of different methods for estimating rainfall excess and base flow on the accuracy of modeling surface runoff (Mays and Taur 1982; Cheng and Wang 2002; Ma et al. 2011). Before the identification of unit hydrographs and component flows from rainfall, evaporation and streamflow data (IHACRES) (Jakeman et al. 1990; Jakeman and Hornberger 1993) and tank (Sugawara 1979; Sugawara 1995; Madsen 2000; Yue and Hashino 2000; Hashino et al. 2002; Chen et al. 2003; Lee and Singh 2005), hydrological UH-based modeling was used to generate the direct runoff component by a linear convolution with the specific input/output structures.

A streamflow hydrograph observed from a discharge site generally consists of surface flow, subsurface flow, and groundwater. Surface flow denotes water stored or flowing on the Earth's surface, subsurface flow is water stored or flowing between the land surface and water table, and groundwater denotes water stored or flowing beneath the water table. Yue and Hashino (2000) defined quick flow as equivalent to surface flow, divided subsurface runoff into rapid and delayed subsurface runoffs, and defined slow flow as the sum of subsurface flow and groundwater. During a heavy rainfall-runoff event, the surface flow is the most important component of a river outlet of a watershed; subsurface and groundwater flows account for relatively little of the streamflow. Large rainfall events usually create large surface runoffs and lead to a high probability of flooding in downstream areas, endangering the lives and property of individuals living there. From this viewpoint, hydrologists typically tend to focus on simulated surface runoffs. Excluding flood disaster, excessive groundwater pumping that causes land subsidence is another familiar problem related to water resources in Taiwan. By understanding better how the components of runoff contribute to streamflow or how the individual runoff components may be quantified, a better understanding of groundwater recharge can be developed, which will help guide the use and allocation of water resources.

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Hydrological modeling usually considers watersheds for linear cascade reservoirs. Hydrologic cascades have been used for decades to conceptually describe the catchment response to excess rainfall. Simple cascade models do not have any physical significance. The output of an upstream reservoir becomes the input to the next reservoir downstream, and the model behavior is governed by two parameters, as in the Nash model (Nash 1957). The IHACRES model, which simulates quick and slow runoffs, has a linear module that allows any configuration of stores in parallel or series. This linear module is a recursive relation at a time step for generating streamflow, calculated as a linear combination of its antecedent values and excess rainfall. In that linear combination, the best configuration of two parallel storages in the linear routing module is frequently used to generate streamflow components. The IHACRES model has several variants, such as catchment moisture deficit (CMD)-IHACRES (Schreider et al. 2002; Evans 2003; Croke et al. 2006; Carcano et al. 2008) and identification of unit hydrographs and component flows (Andréassian et al. 2001). These variants identify separate UHs for relatively quick and slow response components of streamflow, leading to a continuous hydrograph separation. The tank model is represented by a cascade of conceptual tanks, and the whole period is divided into subperiods, the components of which play the main part. The volumes and shapes of these tanks are calculated in each subperiod and are used to adjust the respective tanks. The tank model consists of two types of tanks that can be approximated by a linear model by moving the side outlets or outlet to the bottom. Complex cascade models generally have many parameters related to catchment characteristics. For example, the IHACRES model requires between five and seven parameters to be calibrated, and at least eight parameters are required for the tank model. Linear cascade models have several practical applications in hydrology, including estimating a runoff hydrograph at the catchment outlet.

This study adopts the assumptions of an IUH model, such as Nash-type linear reservoirs-that is, a uniform spatial distribution of rainfall and the principle of linear superposition. The proposed model consists of serial cascades of three linear reservoirs and one in parallel. Each linear reservoir has a kernel function with an exponential expression derived from the equation of continuity and convolution integral. These exponential expressions illustrate the storage statuses of the linear reservoirs during rainfall-runoff processes. The block kriging method was used to estimate the mean rainfall as inputs for the model. This study calibrates model parameters of early and later periods using 34 rainfall-runoff events and tests the efficiency of the model using 18 cases in two periods. Therefore, the first application of the proposed model is generating hydrograph components (quick and slow flows) in a specific river with hydrological serial and parallel cascades during storms. The time to peak (as a time characteristic) and peak discharge and total discharge (as flow characteristics) of runoff components in early and later periods were identified by relating rainfall and streamflow observations. This study completes identifications of shape characteristics of component hydrographs related to recordings of rainfall and streamflow and compares their differences between early and later periods. Results show that the proposed method can rapidly determine hydrographs of runoff components in a rainfall-runoff process and only requires rainfall-streamflow recordings without complex model simulation.

Method of Areal Rainfall Estimation

Excessive surface runoff generally comes from large rainfall events with a high rainfall intensity that exceeds the infiltration rate of the

676 / JOURNAL OF HYDROLOGIC ENGINEERING © ASCE / JUNE 2013

soil surface. Rainfall patterns vary greatly in space and time (Syed et al. 2003; Basistha et al. 2008; Guhathakurta and Rajeevan 2008). Some rain gauges are more important than others because they are spatially representative of rainfall variations; thus, relative weights can be assigned to these rain gauges to calculate an areal average. Areal rainfall computed from these representative sites is usually used to represent rainfall characteristics of that region. Traditional methods, such as the Thiessen polygon method, have been used to compute mean rainfall. For the block kriging method, a semivariogram with a spatial relationship has also been used to describe variation of rainfall processes in space and determine the point or areal rainfall via the block kriging system. The kriging approach has many applications in various research fields, including the design of rain-gauge networks (Bastin et al. 1984; Cheng et al. 2008a), variogram identification (Lebel and Bastin 1985), spatial interpolation of rainfall (Goovaerts 2000; Syed et al. 2003), and space-time rainfall interpolation (Cheng et al. 2007).

The block kriging method was used to estimate hourly spatially uniform rainfall over the whole watershed. The kriging method is theoretically better than the Thiessen method because kriging has a spatial structure (i.e., semivariogram), whereas Thiessen has a lesser ability to represent the spatial structure of rainfall.

Intrinsic Hypothesis

The set of time sequences of discontinuous point-rainfall depths with time period p(t, x) can be considered as a realization of twodimensional random fields. Considering *n* rain gauges in a river basin, for each time period *t*, a realization $\pi(t)$ of the random *n* vectors can be expressed as

$$\pi(t) = [p(t, x_1), p(t, x_2), \dots, p(t, x_n)]$$
(1)

The intrinsic hypothesis for rainfall depth p(t, x) is

$$m(t, x) = E[p(t, x)]$$
(2)

$$\gamma(t, h_{ij}) = \gamma(t, x_i, x_j) = \frac{1}{2} E\{[p(t, x_i) - p(t, x_j)]^2\}$$
(3)

where $\gamma(t, h_{ij})$ = semivariogram of rain gauges x_i and x_j (mm²); and h_{ij} = distance between arbitrary rain gauges x_i and x_j (m). The experimental semivariogram, $\gamma(t, h_{ij})$, of rainfall depth is

$$\gamma(t, h_{ij}) = \frac{1}{2T} \sum_{t=1}^{T} \{ [p(t, x_i) - p(t, x_j)]^2 \}$$
(4)

where T = total duration of all rainfall events (h).

Climatological Mean Semivariogram

Bastin et al. (1984) proposed a basic semivariogram, called the scaled climatological mean semivariogram, which Cheng et al. (2007) later established through dimensionless rainfall data on a project basin. The relationship between the experimental semivariogram and the scaled climatological mean semivariogram is

$$\gamma(t, h_{ij}) = \omega(t)\gamma_d^*(h_{ij}, a) = s^2(t)\gamma_d^*(h_{ij}, a)$$
(5)

where $\omega(t) = \text{sill of semivariogram for time period } t \text{ (mm}^2\text{)}$ and is time variant; $a = \text{range of scaled climatological mean semivario$ gram (m) and is time invariant; and <math>s(t) = standard deviation ofrainfall of all rain gauges for time period t (mm). The basic semivariogram is expressed as

$$\gamma_d^*(h_{ij}, a) = \frac{1}{2T} \sum_{t=1}^T \left\{ \left[\frac{p(t, x_i) - p(t, x_j)}{s(t)} \right]^2 \right\}$$
(6)

The basic experimental semivariogram can be calculated using Eq. (6). This semivariogram is not spatially continuous because it is derived from discontinuous point observations. A realistic application for a block kriging method is to use a popular semivariogram model, called the power model, to obtain spatial continuity of rainfall variations in Taiwan. The equation for the power model is

$$\gamma_d^*(h_{ij}, a) = \omega_0 h_{ij}^a, \qquad a < 2 \tag{7}$$

where $\omega_0 = \text{sill}$ of scaled climatological mean semivariogram (mm²) and is a constant of approximately 1, except for power model.

Block Kriging System

The block kriging method obtains optimal weights by assuming a given spatial structure of rainfall. The system is derived by applying the following Lagrange multipliers:

$$\begin{cases} \sum_{j=1}^{n} \lambda_j \gamma(x_i, x_j) + \mu = \bar{\gamma}(V, x_i), & i = 1, 2, \dots, n \\ \\ \sum_{i=1}^{n} \lambda_i = 1 \end{cases}$$
(8)

$$\sigma_K^2 = \sum_{i=1}^n \lambda_i \bar{\gamma}(V, x_i) + \mu \tag{9}$$

where $\gamma(x_i, x_j)$ = semivariogram of rain gauge x_i and rain gauge x_j (mm²); $\bar{\gamma}(V, x_i)$ = average semivariogram of estimated area V and rain gauge x_i (mm²); λ_i = weighting of each rain gauge; σ_K^2 = kriging estimated variance (mm²); and μ = Lagrange multipliers (mm²).

Block Kriging Estimator

The estimated area V must be divided into M grids before calculating the hourly mean rainfall of storm events over the watershed [Eq. (8)]. Therefore, Eq. (8) can be rewritten as

$$\begin{cases} \sum_{j=1}^{n} \lambda_{j} \gamma(x_{i}, x_{j}) + \mu = \frac{1}{M} \sum_{m=1}^{M} \gamma(V_{m}, x_{i}), & i = 1, 2, \dots, n \\ \sum_{i=1}^{n} \lambda_{i} = 1 \end{cases}$$
(10)

where $V_m = m$ th grid in estimated area. The block kriging method has best linear unbias estimation (BLUE) characteristics in geostatistics. The estimator Z_K^* of the hourly mean rainfall is a linear combination of *n* available point–rainfall recordings $Z(x_i)$ located at x_i and with weightings λ_i . The kriging estimator can be expressed as

$$Z_K^* = \sum_{i=1}^n \lambda_i Z(x_i) \tag{11}$$

Procedure for Calculating Hourly Mean Rainfall

To compute hourly mean rainfall, hourly semivariograms must be determined in advance. This study uses the following procedures to calculate hourly semivariogram $\gamma(t, h_{ii})$:

1. Calculate the values of variance $s^2(t)$ and mean m(t) of rainfall for *n* rain gauges in each time period *t*;

- 2. Apply Eq. (6) to compute the scaled climatological mean semivariogram $\gamma_d^*(h_{ij}, a)$ of all rainfall events and use the power model for fitting to obtain the parameters ω_0 and a; and
- 3. The hourly semivariogram can be obtained using Eq. (5), in which the variance $s^2(t)$ is multiplied by the scaled climato-logical mean semivariogram $\gamma_d^*(h_{ij}, a)$.

The block kriging method is appropriate for estimating mean rainfall. When applied to a lumped/distributed model, the block kriging method can easily calculate the mean rainfall of an entire watershed and its divisions using the following procedures:

- 1. Divide the study watershed into a suitable grid—a suitable grid can be determined through the kriging estimated variance [that is, Eq. (9)];
- 2. Calculate the semivariograms between arbitrary rain gauges x_i and x_i [that is, the $\gamma(x_i, x_i)$ on the left-hand side in Eq. (40)];
- 3. Calculate the mean semivariogram between the grids of the estimated area V and each rain gauge x_i (that is, the $\sum_{m=1}^{M} \gamma(V_m, x_i)/M$ on the right-hand side of Eq. (40); Fig. 1 shows the computation procedure of the mean semivariogram];
- 4. Solve the matrix of Eq. (40) to obtain rain-gauge weightings, and then apply Eq. (11) to calculate the hourly mean rainfall and its estimated variance.

Model

The model in this study is constructed from three linear serial reservoirs with one in parallel. The inputs and outputs of linear systems are analogous to natural flows as runoff components and infiltration. The convolution integral was used to describe the interior transformations of independent systems for the inputs and outputs.

Convolution Integral

The convolution integral is the response of direct runoff at $t - \tau$ time to the complete input time function $I(\tau)$, which can then be derived by integrating the response to the relative constituent impulses:

$$Q(t) = \int_0^t I(\tau)u(t-\tau)d\tau$$
(12)

where τ = dummy variable; $u(t - \tau)$ = kernel function for watershed; $I(\tau)$ = system's input function; and Q(t) = system's output function.

Structure and Flow Mechanism within Model

The proposed model is a lumped rainfall-runoff model for single input and multiple outputs. Average rainfall is the single input for the whole model system, and the outputs are surface, subsurface runoffs, and groundwater at the watershed outlet. Subsurface runoff includes rapid and delayed subsurface runoffs. Rapid subsurface runoff is water flowing into the soil layer near the surface. Delayed subsurface runoff is the flow far away from the infiltration surface. Surface runoff also denotes quick runoff, whereas slow runoff is a sum of subsurface runoff and groundwater runoff. Hence, the structure of the model is three serial linear reservoirs with one in parallel, as derived by Yue and Hashino (2000). Fig. 2 shows the model structure.

The proposed model (Fig. 2) has one horizontal opening and a vertical opening in the upper and middle reservoirs in serial (Tank 1 and Tank 2), whereas the parallel reservoir (Tank 0) and the lowest serial reservoir (Tank 3) only have a horizontal opening. The rates at which water moves through the opening for the horizontal

JOURNAL OF HYDROLOGIC ENGINEERING © ASCE / JUNE 2013 / 677



Fig. 1. Computation of mean semivariogram between estimated area and rain gauges

openings of one parallel and three serial reservoirs are a_0 , a_1 , a_2 , and a_3 , and b_1 and b_2 are for the vertical openings of the upper and middle reservoirs in serial, respectively. Flow discharges q_1 , q_2 , and q_3 of the horizontal openings at the bottom of the three serial reservoirs are modeled as rapid subsurface, delayed subsurface, and groundwater runoffs, respectively. The analogous meaning of surface runoff is indicated by flow q_0 of a horizontal opening of the parallel reservoir when storage in the upper reservoir in serial is higher than the height S_c itself. Height S_c describes the soil antecedent moisture before rainfall. The infiltration amount f_1 flows from a vertical opening in the upper reservoir to the middle reservoir in serial. Discharge f_2 represents the amount of percolation coming from the deep soil aquifer, flowing from the middle reservoir to the lowest reservoir in serial.

Rainfall *r* first falls into the upper reservoir in serial (Tank 1), which begins storing rainwater (i.e., $S_1 > 0$). Rapid subsurface runoff q_1 and infiltration f_1 simultaneously flow out of the upper reservoir in serial. When the storage height of the upper reservoir in serial exceeds height S_c (Tank 1 is full), overflow occurs from the upper reservoir in serial (Tank 1) to the parallel reservoir (Tank 0) and generates surface runoff q_0 (i.e., $S_1 > S_c$). Infiltration f_1 enters the middle reservoir in serial (Tank 2) and is stored, and the delayed

subsurface runoff q_2 and percolation f_2 then start flowing ($S_2 > 0$). Finally, percolation f_2 flows into the lowest reservoir in serial (Tank 3), and its storage status is the same as that in the middle reservoir in serial; groundwater q_3 flows away from the lowest reservoir in serial.

Storage Functions over Time

According to the flow mechanism of the model, runoff components q_1 , q_2 , q_3 , infiltration f_1 , and percolation f_2 are storage functions of three reservoirs in serial, whereas surface runoff q_0 is the amount that overflows from the upper reservoir in serial and then flows into the parallel reservoir and out from its horizontal opening. These outflows, except surface runoff q_0 , are expressed as follows:

$$q_i(t) = a_i S_i(t), \qquad i = 1, 2, 3(\text{mm/h})$$
 (13)

$$f_i(t) = b_i S_i(t), \qquad i = 1, 2(\text{mm/h})$$
 (14)

Each reservoir in this study is an independent input–output system that satisfies the equation of continuity [Eq. (15)]:



Fig. 2. Model structure of three serial reservoirs with one in parallel

By combining the equation of continuity and the convolution integral, the storage functions can be separately obtained from specific inputs of three serial reservoirs and a parallel reservoir. The unit input of the upper reservoir in serial (Tank 1) is the rainfall occurring between 0 and Δt (the value of Δt depends on the recording interval for the rainfall data; in this study, it is 1 h), and that of the other time periods is zero. Hence, instantaneous input $I_1(t)$ equals $1/\Delta t$, and $C_1 = a_1 + b_1$; thus, the storage height $S_1(t)$, which is less than S_c , of the unit input for the upper reservoir in serial (Tank 1) can be derived as follows:

$$S_1(t) = \frac{1}{\Delta t} \frac{(1 - e^{-C_1 t})}{C_1}, \qquad 0 < t \le \Delta t$$
(16)

$$S_1(t) = \frac{1}{\Delta t} \frac{(e^{C_1 \Delta t} - 1)e^{-C_1 t}}{C_1}, \qquad t > \Delta t$$
(17)

Similarly, unit input $I_2(t)$ of the middle reservoir in serial (Tank 2) is the infiltration output $f_1(t)$ of the upper reservoir in serial [i.e., $I_2(t) = f_1(t) = b_1S_1(t)$] and $C_2 = a_2 + b_2$. Thus, the storage height $S_2(t)$ of the unit input for the middle reservoir in serial (Tank 2) is

$$S_{2}(t) = \frac{1}{\Delta t} \frac{b_{1}}{C_{1}C_{2}} \left[1 + \frac{C_{2}}{C_{1} - C_{2}} e^{-C_{1}t} - \frac{C_{1}}{C_{1} - C_{2}} e^{-C_{2}t} \right],$$

0 < t < \Delta t (18)

$$S_{2}(t) = \frac{1}{\Delta t} \frac{b_{1}}{C_{1}C_{2}} \left[\frac{-C_{2}(e^{C_{1}\Delta t} - 1)}{C_{1} - C_{2}} e^{-C_{1}t} + \frac{C_{1}(e^{C_{2}\Delta t} - 1)}{C_{1} - C_{2}} e^{-C_{2}t} \right],$$

$$t > \Delta t$$
(19)

Finally, the unit input of the lowest reservoir in serial (Tank 3) is $I_3(t) = f_2(t) = b_2S_2(t)$ and $C_3 = a_3$. Thus, the mathematical expression of storage height $S_3(t)$ of the lowest reservoir in serial (Tank 3) is

$$S_{3}(t) = \frac{1}{\Delta t} \frac{b_{1}b_{2}}{C_{1}C_{2}a_{3}} \left[1 - \frac{C_{2}a_{3}}{(C_{1} - C_{2})(C_{1} - a_{3})} e^{-C_{1}t} + \frac{C_{1}a_{3}}{(C_{1} - C_{2})(C_{2} - a_{3})} e^{-C_{2}t} - \frac{C_{1}C_{2}}{(C_{1} - a_{3})(C_{2} - a_{3})} e^{-a_{3}t} \right],$$

$$0 < t \le \Delta t$$
(20)

$$S_{3}(t) = \frac{1}{\Delta t} \frac{b_{1}b_{2}}{C_{1}C_{2}a_{3}} \left[\frac{C_{2}a_{3}(e^{C_{1}\Delta t} - 1)}{(C_{1} - C_{2})(C_{1} - a_{3})} e^{-C_{1}t} - \frac{C_{1}a_{3}(e^{C_{2}\Delta t} - 1)}{(C_{1} - C_{2})(C_{2} - a_{3})} e^{-C_{2}t} - \frac{C_{1}C_{2}(e^{a_{3}\Delta t} - 1)}{(C_{1} - a_{3})(C_{2} - a_{3})} e^{-a_{3}t} \right],$$

$$t > \Delta t$$
(21)

Similar to the upper reservoir in serial (Tank1), for the unit input $(I_0 = 1)$ in duration Δt , the unit pulse–response function of the parallel reservoir (Tank 0), which is used to generate surface runoff q_0 , can be obtained as

$$u_0(t) = \frac{1 - e^{-a_0 t}}{\Delta t}, \qquad 0 < t \le \Delta t$$
 (22)

$$u_0(t) = \frac{1}{\Delta t} (e^{a_0 \Delta t} - 1) e^{-a_0 t}, \qquad t > \Delta t$$
(23)

Parameter Limitations

On the basis of the physical significance of the principles of the hydrological cycle, soil infiltration, and runoff generation, the model parameters should be confined to the following eight limitations:

- 1. For $a_0 > a_1$, the rate coefficient of surface runoff q_0 must be larger than that of rapid subsurface runoff q_1 ;
- 2. For $a_1 \ge a_2$, the rate coefficient of rapid subsurface runoff q_1 must be larger than that of delayed subsurface runoff q_2 ;
- 3. For $a_2 > a_3$, the rate coefficient of delayed subsurface runoff q_2 must be larger than groundwater amount q_3 ;
- 4. For $b_1 > b_2$, the rate coefficient of infiltration f_1 resulting from the upper reservoir must be larger than that of percolation f_2 coming from the middle reservoir;
- 5. For $1 (a_1 + b_1) \ge 0$, the sum of two opening ratios in the upper reservoir in serial must be less than or equal to 1;
- 6. For $1 (a_2 + b_2) \ge 0$, the sum of two opening ratios in the middle reservoir in serial must be less than or equal to 1;
- 7. For $1 a_3 \ge 0$, the opening ratio of the lowest reservoir in serial must be less than or equal to 1; and
- 8. For $1 a_0 \ge 0$, the opening ratio of the parallel reservoir must be less than or equal to 1.

Parameter Optimization and Evaluation Criteria

Objective Function

An objective function must be assigned to optimize system parameters. This objective function can be used minimize the error between simulations and observations of the runoff hydrographs. This study uses the following expression (Yue and Hashino 2000) for parameter optimization:

$$F_{\rm obj} = \frac{1}{T} \sum_{t=1}^{T} \left\{ \frac{[Q_{\rm obs}(t) - Q_{\rm est}(t)]^2}{Q_{\rm obs}(t)} \right\}$$
(24)

where F_{obj} = value of objective function; T = total duration of observed hydrograph; $Q_{obs}(t)$ = observed value of runoff hydrograph at time period t; and $Q_{est}(t)$ = simulated value at time period t.

Evaluation Criteria

This study uses three criteria to evaluate the suitability of the rainfall-runoff model for the basin of interest: the coefficient of efficiency (CE) (Nash and Sutcliffe 1970; Nayak et al. 2005), error of peak discharge (EQ_p) , and error of the time for peak to arrive (ET_p) . The coefficient of efficiency is commonly used as a measure of model performance.

Peak discharge and time to peak are also important characteristics of flood hydrographs. Hence, this study also examines the differences in peak quantity and time to peak between observations and simulations. The error of peak discharge and error of time for peak to arrive are also frequently used to examine the simulated results (Chen et al. 2003; Moramarco et al. 2005). These three criteria are as follows:

1. The coefficient of efficiency, CE, is defined as

$$CE = 1 - \frac{\sum_{t=1}^{T} [Q_{est}(t) - Q_{obs}(t)]^2}{\sum_{t=1}^{T} [Q_{obs}(t) - \bar{Q}_{obs}(t)]^2}$$
(25)

JOURNAL OF HYDROLOGIC ENGINEERING © ASCE / JUNE 2013 / 679

where $Q_{est}(t)$ = discharge of simulated hydrograph for time period t; $Q_{obs}(t)$ = discharge of observed hydrograph for time period t; and $\bar{Q}_{obs}(t)$ = average discharge of observed hydrograph for time period t. The better the fit, the closer CE is to one. A negative value for CE means that model predictions are worse than predictions using a constant that is equal to the average observed value.

2. The error of peak discharge, $EQ_p(\%)$, is defined as

$$EQ_p(\%) = \frac{Q_{\text{est},p} - Q_{\text{obs},p}}{Q_{\text{obs},p}} \times 100\%$$
(26)

where $Q_{\text{est},p}$ = peak discharge of simulated hydrograph; and $Q_{\text{obs},p}$ = peak discharge of observed hydrograph.

3. The error of the time for peak to arrive, ET_p , is defined as

$$ET_p = T_{\text{est},p} - T_{\text{obs},p} \tag{27}$$

where $T_{\text{est},p}$ = time for simulated hydrograph peak to arrive; and $T_{\text{obs},p}$ = time required for observed hydrograph peak to arrive.

Watershed Description

Geographical Features

The Wu-Tu Watershed was chosen as a test site to explore the characteristics of runoff components resulting from the proposed model of three serial cascade reservoirs with a parallel reservoir. This watershed is an area located upstream of the Wu-Tu discharge site in the Kee-Lung Watershed (Fig. 3). The Kee-Lung River is one of three tributaries of the Tamshui River Basin (Fig. 3). The other tributaries are Da-Han Stream and Hsin-Tien Stream. The Wu-Tu Watershed covers approximately 204 km², and the mean annual precipitation and runoff depth are 2,865 and 2,177 mm, respectively. Because of the rugged topography of the watershed, runoff pathlines are short and steep, and rainfall is not uniform in either time or space. Large floods arrive rapidly in the middle to downstream reaches of the watershed, causing serious damage during summers.

Data Studied

The Wu-Tu Watershed has three rain gauges (Jui-Fang, Wu-Tu, and Huo-Shao-Liao) and one discharge site (Wu-Tu). The study sample consisted of 52 rainfall-runoff events recorded from 1966-2008. In total, 34 cases were selected for parameter calibration, and the remaining 18 events were used to verify the applicability of the calibrated parameters. Because the event data were recorded across 42 years, this study uniformly separates the events from two time periods and then compares the differences in the shape characteristics of their component hydrographs. The two time periods are the early period (1966-1993) and later period (1994-2008). Because three rain gauges on the Wu-Tu Watershed only produce three pairs, it is not possible to establish a basic semivariogram of hour rainfall (because of insufficient data). Therefore, this study extends the number of rain gauges to 14 gauges in the Tamshui River Basin, as shown in Fig. 3. The scaled climatological mean semivariogram of rainfall was analyzed using hourly data from 14 rain gauges on the Tamshui River Basin, which also included three rain gauges (Jui-Fang, Wu-Tu, and Huo-Shao-Liao) from the Wu-Tu Watershed. Because the other 11 gauges are not located in the Wu-Tu Watershed, hourly inputs of mean rainfall for the model were estimated using the climatological mean semivariograms and Kriging system derived from hourly data of three rain gauges (i.e., Jui-Fang, Wu-Tu, and Huo-Shao-Liao) located in the Wu-Tu Watershed.

Results and Discussion

This study defines four hydrograph components, but five components must be determined. The previous four components (surface runoff, rapid subsurface runoff, delayed subsurface runoff, and groundwater) are produced from a current rainfall event and can



Fig. 3. Location maps of Tamshui River Basin and Wu-Tu Watershed

be called new discharge. The last component is base flow, which is not a newly generated groundwater flow in the current rainfallrunoff event and can therefore be called old discharge. This study considers base flow a constant, and its discharge value is the lowest discharge of a rising limb in a streamflow hydrograph. Model parameters were determined by applying an optimization method to rainfall and streamflow data. Horizontal outflows of three linear serial reservoirs with a reservoir in parallel generate surface runoff, rapid subsurface runoff, delayed subsurface runoff, and groundwater. Finally, the shape characteristics of hydrographs were identified by relating simulated components to rainfall and streamflow observations.

Hourly Mean Rainfall

The hourly semivariogram is a function of time t, isotropy, and a time average form with a nonzero and T time interval. The analytical results of the scaled climatological mean semivariogram were completed using the 52 rainfall events recorded by 14 rain gauges in or around the watershed. The power form (Fig. 4) was then applied for fitting as follows:

$$\gamma_d^*(h_{ij}, a) = \omega_0 h^a = 0.093 h^{0.243}, \qquad R^2 = 0.906$$
 (28)

where ω_0 = scaled parameter of scaled climatological mean semivariogram (mm²). Variance $s^2(t)$ of a realization $\pi(t)$ for each time period *t* can be easily calculated from the hourly rainfall measurements. Hourly semivariograms of rainfall events can then be directly calculated using Eqs. (5) and (28).

The estimated area must be divided into *M* grids before calculating the hourly mean rainfall during storm events over the watershed by applying Eq. (40). The estimated area was divided into $2,665 \times 1 \text{ km}^2$ grids. This study uses observations from three rain gauges located in the Wu-Tu Watershed to estimate the hourly mean rainfall.

Parameter Calibration

The simulated runoff components were exported from the model system. In the process of translating the rainfall runoff, the model parameters for each event were determined using the shuffled complex evolution (SCE) optimal algorithm (Duan et al. 1993). These

calibrated parameters reflect the complex rainfall-runoff processes resulting from the watershed and meteorological characteristics of each case. They also reflect errors in the rainfall estimates, initial conditions, and observed flow. Table 1 and Fig. 5 compare the simulated and observed runoff hydrographs using the three criteria (CE, EQ_p , and ET_p) in two periods.

For calibrated events in early and later periods, regarding the CE criterion, 15 and 11 calibrated events exceed 0.8, two and four cases are within the intervals of 0.7–0.8, and only two cases in the later period are below 0.7 (Table 1). With regard to EQ_n , all samples are smaller than 25% except for two events and two storms in the two time periods. The ET_p values are all less than or equal to 3 h; three in the later period are longer than 3 h. The combined average values of the three criteria (CE, EQ_p , and ET_p) in two time periods are 0.90 and 0.83, 4.60 and -0.37%, and 0.35 and 1.94 h, respectively. The calibration results reveal small differences between three evaluation criteria in early and later periods and also exhibit that the calibration is satisfactory for regenerating rainfall-runoff processes. Model calibration using the three evaluation criteria demonstrates that the calibrated parameters are able to illustrate the situation of the studied watershed during rainfall-runoff process.

Model Verification

The remaining 18 events were used to verify the usability of the proposed model in the early and later periods. The mechanism for translating rainfall into watershed runoffs is the same as the previous calibration process. The calibrated parameters of 34 events were separately averaged according to the two divided time periods. Table 2 lists the mean values of the seven parameters (S_c , a_0 , a_1 , a_2 , a_3 , b_1 , and b_2) in the proposed model. These values represent the characteristics of antecedent soil condition, four runoff components, infiltration, and percolation in two time periods, respectively. Because the hourly value of antecedent soil moisture is difficult to measure or approximate, this study assumes that the value is a constant for the two periods. Table 3 and Fig. 6 present the acceptable verification results.

The values of the coefficient of efficiency for the model verification are equal to or exceed 0.70, excluding those for three events in the early period and for two cases in the later period. The error of



Fig. 4. Scaled climatological mean semivariogram in Wu-Tu Watershed

Table 1. Calibration Results for Model of Three Serial Reservoirs with One Parallel Reservoir in Early and Later Periods

	Early period			L	ater period		
Event name (time)	CE	EQ_p (%)	ET_p (h)	Event name (time)	CE	EQ_p (%)	ET_p (h)
CORA (Sep. 6, 1966)	0.95	15.47	0	Storm (Jun. 18, 1994)	0.85	-16.67	0
BETTY (Aug. 16, 1972)	0.97	-0.88	-1	FRED (Aug. 20, 1994)	0.93	15.31	2
BILLIE (Aug. 9, 1976)	0.98	-9.01	1	GLADYS (Sep. 1, 1994)	0.91	-17.05	2
VERA (Jul. 31, 1977)	0.81	33.86	0	HERB (Jul. 31, 1996)	0.95	6.23	1
IRVING (Aug. 14, 1979)	0.94	18.97	0	Storm (Sep. 26, 1999)	0.82	-11.56	4
Storm (Nov. 19, 1980)	0.87	18.61	0	BEBINCA (Nov. 8, 2000)	0.68	12.79	0
ANDY (Jul. 29, 1982)	0.77	15.90	0	RANANIM (Aug. 12, 2004)	0.61	-8.16	1
CECIL (Aug. 9, 1982)	0.80	17.71	0	Storm (Aug. 17, 2004)	0.87	-13.36	2
Storm (Oct. 12, 1983)	0.96	7.54	1	Storm (Oct. 1, 2004)	0.79	12.78	2
Storm (Oct. 14, 1983)	0.99	-5.74	0	Storm (Sep. 5, 2005)	0.78	26.76	1
Storm (Jun. 2, 1984)	0.94	-29.21	0	Storm (Sep. 10, 2005)	0.76	-4.62	3
Storm (Nov. 18, 1984)	0.85	13.01	2	Storm (Dec. 4, 2005)	0.88	1.44	3
NELSON (Aug. 22, 1985)	0.91	-2.56	0	Storm (Dec. 11, 2005)	0.70	17.90	5
ALEX (Jul. 27, 1987)	0.88	-5.92	2	Storm (Jun. 6, 2006)	0.85	10.57	4
ABE (Aug. 30, 1990)	0.76	11.36	-1	Storm (Jun. 15, 2007)	0.94	-28.79	1
Storm (Sep. 1, 1990)	0.92	-4.66	0	Storm (Sep. 4, 2007)	0.94	-18.36	1
Storm (Sep. 2, 1990)	0.92	-16.23	2	Storm (Oct. 10, 2008)	0.87	8.55	1
Mean	0.90	4.60	0.35	Mean	0.83	-0.37	1.94

peak discharge is less than 30% when excluding four and three events in the two periods. Except for one case in the early period and two cases in the later period, the error in the arrival time of the peak for all examined events is 4 h or less. The combined average values of the three criteria (CE, EQ_p , and ET_p) in two time periods are 0.68 and 0.74, -13.04 and -9.92%, and -1.56 and -2.11 h, respectively. The differences of three criteria for verification results between two time periods are similar to those for calibration results. These comparison results of calibration and verification in early and later periods indicate that the observations and simulations have an acceptable goodness of fit, and the model outputs are suitable for further applications.

Time Characteristics of Runoff Components

Clarifying the time characteristics of runoff components is vital to determining the time required to produce maximum discharge. Thus, this study first addresses the time characteristics of hydrographs for time to peak. The time characteristic for time to peak significantly influences the shape of the resulting hydrograph, especially for a quick/surface flow considering flood disaster mitigation.

Table 4 lists the 34 comparison results of times to peak of runoff components and streamflow observations. A review of the simulation results (bold values for 12 cases) of zero ET_p values in Table 1 reveals that the time to peak of a quick runoff hydrograph equals that of the streamflow hydrograph and is earlier than that of the slow runoff hydrograph in the same rainfall-runoff case. The differences in time to peak between slow and quick/streamflow hydrographs range from 1-19 h. Moreover, times to peak for hydrograph components typically occur after the occurrence time of rainfall peak. Times to peak of quick/streamflow hydrographs for 12 events of zero ET_p results occur 2–24 h after peak times of rainfall, and a range of 4-36 h for slow hydrographs. This is because the flow mechanism of a slow response is more complicated than that of a quick (surface) response. Thus, the times to peak of quick flows to peak times of rainfall are smaller than times to peak of slow flows to peak times of rainfall.

This study attempts to infer the relationships between time to peak of hydrograph components and peak time of hyetographs. Because the time differences of various events are large, this study computes the nature logarithmic values of peak time of rainfall and the time to peak of component hydrographs to obtain two ratios of logarithmic values. These two ratios represent the relationships of times to peak of quick and slow flows to peak time of rainfall, respectively. Table 4 (bottom five rows) shows three statistics of ratio values for 17 individual events in early and later periods. A comparison of these data shows that the coefficient of variation in the later period is larger than that in the early period. This suggests that the differences in peak times in the later period varied more than those in the early period. This study also obtains two linear regression results of logarithmic values of times to peak of quick and slow flows to peak time of rainfall, respectively, as follows:

$$\ln(T_{p,Qq}) = 0.528 \times \ln(T_{p,R}) + 1.715,$$

 $R^2 = 0.78$ for the early period (29)

$$\ln(T_{p,Qs}) = 0.216 \times \ln(T_{p,R}) + 2.796,$$

$$R^2 = 0.21 \text{ for the early period}$$
(30)

$$\ln(T_{p,Qq}) = 0.327 \times \ln(T_{p,R}) + 2.425,$$

 $R^2 = 0.58$ for the later period (31)

$$\ln(T_{p,Qs}) = 0.280 \times \ln(T_{p,R}) + 2.680,$$

 $R^2 = 0.52$ for the later period (32)

where the $T_{p,R}$ symbol represents peak time of rainfall observation; the $T_{p,Qs}$ symbol is time to peak of simulated slow flow and the $T_{p,Qq}$ symbol define time to peak of simulated quick flow. These regression results reveal that only the poor R^2 value results from times to peak of slow flows to peak times of hyetographs in early period; those of other three regression results are larger than 0.5. The regression results for times to peak in slow flows to peak times of hyetographs in both periods are similar, but show differences for quick flow in two time periods. However, the times to peak for both component hydrographs may be positively related to the peak time of rainfall with logarithm forms.

Flow Characteristics of Runoff Components

Identifying the flow characteristics of a hydrograph is an essential task in designing hydraulic structures. This study analyzes the



Table 2. Averaged Seven Parameters of Model of Three Serial Reservoirs

 with One Parallel Reservoir in Early and Later Periods

Time classification	S_c	a_0	a_1	a_2	<i>a</i> ₃	b_1	b_2
Early period	95.85	0.4181	0.0465	0.0157	0.0071	0.0465	0.0153
Later period	78.49	0.2501	0.0443	0.0011	0.0003	0.0443	

correlations between streamflow hydrographs and their runoff components for peak discharge and total discharge of flow characteristics. Table 5 lists the hydrograph characteristics of slow and quick simulations along with streamflow observations for 34 calibrated events. This table also shows the relationships among peak discharges and total discharges of the simulated slow, quick, and observed streamflow hydrographs.

Theoretically, the peak discharge of a slow runoff should be smaller than that of a quick runoff and total flows in the same event. The second through fourth and sixth through eighth columns of Table 5 confirm that a large streamflow hydrograph has a large peak for quick runoff, whereas the peak of a slow runoff is small compared with that of streamflow (bold values for 30 cases represent simulation results of events with EQ_p values in between $\pm 20\%$. Using an analytical procedure similar to that used for the time characteristics of runoff components, this study also examines four ratios for peak discharges of runoff components to streamflow observation. In Table 5 (bottom seven rows), the ratios of slow runoffs to streamflow observations in early and later periods spread over

Table 3. Verification Results for Model of Three Serial Reservoirs with One Parallel Reservoir in Early and Later Periods

	Early period	1		Later period				
Event name (time)	CE	EQ_p (%)	ET_p (h)	Event name (time)	CE	EQ_p (%)	ET_p (h)	
FRAN (Sep. 5, 1970)	0.70	37.62	-3	SETH (Oct. 9, 1994)	0.83	33.21	1	
Storm (Oct. 28, 1974)	0.60	-34.88	3	KAI-TAK (Jul. 8, 2000)	0.74	-33.72	-1	
Storm (Feb. 8, 1985)	0.70	-26.72	-3	LEKIMA (Sep. 25, 2001)	0.77	12.50	0	
SARAH (Sep. 1, 1989)	0.60	-34.37	-1	Storm (Mar. 26, 2004)	0.80	-9.80	-3	
OFELIA (Jun. 22, 1990)	0.66	-37.76	-4	Storm (Dec. 23, 2004)	0.76	-18.90	-4	
YANCY (Aug. 19, 1990)	0.70	21.23	-1	LONGWANG (Oct. 1, 2005)	0.67	-24.33	-3	
NAT (Sep. 29, 1991)	0.79	-25.22	-2	BILIS (Jul. 13, 2006)	0.72	-25.62	-3	
Storm (Aug. 29, 1992)	0.70	-10.68	-3	Storm (Mar. 6, 2007)	0.72	-35.58	-4	
Strom (Jun. 5, 1993)	0.71	-6.54	0	Storm (Nov. 3, 2008)	0.67	12.92	-2	
Mean	0.68	-13.04	-1.56	Mean	0.74	-9.92	-2.11	



684 / JOURNAL OF HYDROLOGIC ENGINEERING © ASCE / JUNE 2013

Table 4. Ratio Comparisons of Logarithmic Peak Time between Component Hydrographs and Streamflow in Early and Later Periods

Early period					Later period				
	Time to peak (h)					Time to peak (h)			
Event name (time)	$T_{p,R}$	$T_{p,Qs}$	$T_{p,Qq}$	$T_{p,Qobs}$	Event name (time)	$T_{p,R}$	$T_{p,Qs}$	$T_{p,Qq}$	$T_{p,Qobs}$
CORA (Sep. 6, 1966)	15	24	22	22	Storm (Jun. 18, 1994)	14	24	19	19
BETTY (Aug. 16, 1972)	28	34	32	33	FRED (Aug. 20, 1994)	34	44	41	39
BILLIE (Aug. 9, 1976)	14	25	21	20	GLADYS (Sep. 1, 1994)	12	25	19	17
VERA (Jul. 31, 1977)	19	28	24	24	HERB (Jul. 31, 1996)	17	30	28	27
IRVING (Aug. 14, 1979)	39	43	41	41	Storm (Sep. 26, 1999)	18	35	31	27
Storm (Nov. 19, 1980)	20	35	32	32	BEBINCA (Nov. 8, 2000)	6	39	30	30
ANDY (Jul. 29, 1982)	9	26	19	19	RANANIM (Aug. 12, 2004)	2	19	15	14
CECIL (Aug. 9, 1982)	5	33	14	14	Storm (Aug. 17, 2004)	5	20	17	15
Storm (Oct. 12, 1983)	13	24	20	19	Storm (Oct. 1, 2004)	18	33	30	28
Storm (Oct. 14, 1983)	15	25	21	21	Storm (May 9, 2005)	18	41	39	38
Storm (Jun. 2, 1984)	31	40	35	35	Storm (Sep. 10, 2005)	24	40	37	34
Storm (Nov. 18, 1984)	13	49	35	33	Storm (Dec. 4, 2005)	2	17	14	11
NELSON (Aug. 22, 1985)	21	31	24	24	Storm (Dec. 11, 2005)	21	33	29	24
ALEX (Jul. 27, 1987)	7	18	14	12	Storm (Jun. 6, 2006)	22	41	39	35
ABE (Aug. 30, 1990)	18	23	22	23	Storm (Jun. 15, 2007)	15	25	21	20
Storm (Sep. 1, 1990)	8	28	15	15	Storm (Sep. 4, 2007)	38	49	45	44
Storm (Sep. 2, 1990)	20	31	27	25	Storm (Oct. 10, 2008)	9	13	12	11
Statistics	$\ln T_{p,Q_s}$	$/\ln T_{p,R}$	$\ln T_{p,Q_a}$	$/\ln T_{p,R}$	Statistics	$\ln T_{p,Q_s}$	$/\ln T_{p,R}$	$\ln T_{p,Q_a}$	$/\ln T_{p,R}$
Maximum	2.	173	1.	640	Maximum	4.248 3.90		907	
Minimum	1.0	027	1.	014	Minimum	1.070 1.04		046	
Mean	1.2	289	1.	185	Mean	1.628 1.5		545	
Standard deviation	0.2	288	0.	167	Standard deviation	0.991		0.	901
Coefficient of variation	0.2	224	0.	141	Coefficient of Variation	0.609 0.58		583	

Note: Bold values represent simulation results of events are zero ET_p values in Table 1. $T_{p,R}$ = peak time of rainfall observation; $T_{p,Qs}$ = time to peak of simulated slow flow; $T_{p,Qq}$ = time to peak of simulated quick flow; and $T_{p,Qobs}$ = time to peak of streamflow observation.

small ranges of 0.04–0.288 and 0.077–0.363. However, Table 5 shows large degrees of 0.634–1.116 and 0.601–0.975 for quick runoffs to streamflow observations in early and later periods. These comparisons show that the peaks of quick runoffs are slightly

smaller than those of stream flows, and the peaks of slow runoffs are significantly smaller than those of stream flows. Table 5 also shows the four relationships are stably quasi-linear correlation based on four small coefficients of variation. Therefore, this study

Table 5. Direct Ratio Comparisons of Peak Discharges between Component Hydrographs and Streamflow in Early and Later Periods

Early period					Later period				
Peak discharge (m ³ /s)					Peak discharge (m ³ /s			(m^3/s)	
Event name (time)	$Q_{p,s}$	$Q_{p,q}$	$Q_{p,\mathrm{obs}}$		Event name (time)	Ç	$Q_{p,s}$	$Q_{p,q}$	$Q_{p,\mathrm{obs}}$
CORA (Sep. 6, 1966)	143.7	778.9	827		Storm (Jun. 18, 1994)	ç	9.5	340.6	532
BETTY (Aug. 16, 1972)	166.9	510.6	708		FRED (Aug. 20, 1994)	3	9.6	245.8	252
BILLIE (Aug. 9, 1976)	23.2	212.4	260		GLADYS (Sep. 1, 1994)	3	6.2	324.6	439
VERA (Jul. 31, 1977)	145.9	846.3	758		HERB (Jul. 31, 1996)	21	5.4	938.1	1,090
IRVING (Aug. 14, 1979)	259.7	906.5	1,030		Storm (Sep. 26, 1999)		7.4	59.3	80
Storm (Nov. 19, 1980)	125.6	734.9	765		BEBINCA (Nov. 8, 2000)	11	1.0	551.3	632
ANDY (Jul. 29, 1982)	67.4	340.1	364		RANANIM (Aug. 12, 2004)	1	2.6	80.0	102
CECIL (Aug. 9, 1982)	146.3	607.1	682		Storm (Aug. 17, 2004)		4.1	21.4	30
Storm (Oct. 12, 1983)	72.9	638.0	670		Storm (Oct. 1, 2004)	1	1.7	60.0	69
Storm (Oct. 14, 1983)	100.5	792.7	980		Storm (May 9, 2005)	8	0.7	237.1	256
Storm (Jun. 2, 1984)	96.3	900.1	1,420		Storm (Sep. 10, 2005)	1	2.5	77.3	99
Storm (Nov. 18, 1984)	98.6	349.7	401		Storm (Dec. 4, 2005)		6.8	37.0	48
NELSON (Aug. 22, 1985)	107.7	1,108.4	1,250		Storm (Dec. 11, 2005)	2	4.3	132.6	169
ALEX (Jul. 27, 1987)	61.8	431.4	527		Storm (Jun. 6, 2006)	2	1.3	94.7	112
ABE (Aug. 30, 1990)	226.9	629.3	789		Storm (Jun. 15, 2007)	2	2.8	178.5	297
Storm (Sep. 1, 1990)	13.0	268.0	327		Storm (Sep. 4, 2007)	3	5.6	229.6	327
Storm (Sep. 2, 1990)	124.7	573.8	857		Storm (Oct. 10, 2008)	17	9.1	320.2	494
Statistics	$Q_{p,s}$	$Q_{p,obs}$	$Q_{p,q}/Q_{p,obs}$		Statistics		Q_{p}	$_{s}/Q_{p,\rm obs}$	$Q_{p,q}/Q_{p,obs}$
Maximum	0.	288	1.116		Maximum		(0.363	0.975
Minimum	0.	040	0.634		Minimum		(0.077	0.601
Mean	0.	159	0.854		Mean		(0.171	0.780
Standard deviation	0.	073	0.116		Standard deviation		(0.079	0.103
Coefficient of variation	0.	455	0.136		Coefficient of Variation		(0.463	0.132

Note: Bold values represent simulation results of events whose EQ_p values are in between $\pm 20\%$. $Q_{p,q}$ = peak discharge of simulated quick flow; $Q_{p,s}$ = peak discharge of simulated slow flow; and $Q_{p,obs}$ = peak discharge of streamflow observation.

obtains four linear regression results resulting from relation ratios of peak discharges for relating hydrograph components to observed streamflow.

$$Q_{p,q} = 0.725 \times Q_{p,\text{obs}} + 87.043, \ R^2 = 0.83$$
 for the early period (33)

 $Q_{p,s} = 0.095 \times Q_{p,\text{obs}} + 45.875, \ R^2 = 0.22$ for the early period (34)

 $Q_{p,q} = 0.821 \times Q_{p,\text{obs}} - 11.908, \ R^2 = 0.97$ for the later period (35)

 $Q_{p,s} = 0.203 \times Q_{p,obs} - 4.65, R^2 = 0.80$ for the later period (36)

where the $Q_{p,q}$ symbol represents peak discharge of simulated quick flow; the $Q_{p,s}$ symbol denotes peak discharge of simulated slow flow, and the $Q_{p,obs}$ symbol is peak discharge of streamflow observation. The R^2 values of regression results are similar to those of previous regression analyses for time to peak of time characteristics. Three R^2 results are larger than 0.8, whereas only one result, which is based on peak discharge of slow flows to those of streamflow observations, is poor. These regression results have enough representativeness to reveal linear correlations of peak discharges between hydrograph components and observed streamflow. These linear correlations reveal little difference between the direct ratios of peak discharges in early and later periods. The coefficient of variation and R^2 value also show that correlations of peak discharges resulting from quick flow and streamflow are stronger than those based on slow flow and streamflow.

Table 6 shows the total discharges of simulated slow and quick hydrographs and observation hydrographs of streamflow for

34 calibrations. In a typhoon or large storm, the total discharge of a quick runoff should exceed that of a slow runoff in a rainfall-runoff process. The second through fourth and sixth through eighth columns in Table 6 present agreeable results for all cases (bold values for 26 cases represent simulation results of events with CE values larger than or equal to 0.8.). Using the same procedure as that used to analyze peak discharge, this study calculates four ratios for the total discharges of runoff components to streamflow observation. Computation results reveal that total discharges of slow flows are smaller than those of quick runoffs based on ratio percentages: 11.2-47.8% and 26.2-49.6% for slow flows, and 47.8-77.7% and 42.0-62.6% for quick flows in the early and later periods, respectively. These results suggest that the total discharge of a slow runoff is always smaller than that of a quick runoff in a rainfall-runoff process. The four small coefficients of variation reveal that four direct ratios for total discharges between simulated hydrograph components and streamflow observations should have strong and stable linear correlations. Based on the well values of coefficients of variation, four linear regression analyses were completed by relating total discharges of runoff components to those of streamflow observations. These values are presented as below

$$Q_q = 0.578 \times Q_{obs} + 64.911, \ R^2 = 0.90$$
 for the early period (37)

$$Q_s = 0.357 \times Q_{obs} - 24.909, \ R^2 = 0.80$$
 for the early period (38)

$$Q_q = 0.541 \times Q_{obs} + 1.172, \ R^2 = 0.97$$
 for the later period (39)

 $Q_s = 0.409 \times Q_{obs} - 132.824, \ R^2 = 0.93$ for the later period (40)

Table 6. Direct Ratio Comparisons of Total Discharges between Component Hydrographs and Streamflow in Early and Later Periods

]	Early period			Later period				
	Total	discharge (1	m^3/s)		Total discharge (m ³ /s)			
Event name (time)	Q_s	Q_q	$Q_{\rm obs}$	Event name (time)	Q_s	Q_q	$Q_{ m obs}$	
CORA (Sep. 6, 1966)	3,476.2	4,967.0	8,854	Storm (Jun. 18, 1994)	2,498.3	3,118.7	5,321	
BETTY (Aug. 16, 1972)	4,409.2	7,679.9	13,489	FRED (Aug. 20, 1994)	1,854.1	2,345.8	4,587	
BILLIE (Aug. 9, 1976)	948.2	1,893.0	2,768	GLADYS (Sep. 1, 1994)	2,538.3	3,058.8	5,969	
VERA (Jul. 31, 1977)	4,243.7	5,841.2	8,877	HERB (Jul. 31, 1996)	7,285.4	7,715.0	15,498	
IRVING (Aug. 14, 1979)	5,946.9	10,150.0	17,397	Storm (Sep. 26, 1999)	527.2	880.9	1,807	
Storm (Nov. 19, 1980)	3,019.0	5,228.0	10,514	BEBINCA (Nov. 8, 2000)	5,100.5	8,855.7	15,762	
ANDY (Jul. 28, 1982)	2,319.2	2,998.6	5,407	RANANIM (Aug. 12, 2004)	527.1	611.0	1,455	
CECIL (Aug. 9, 1982)	5,327.8	7,698.9	13,197	Storm (Aug. 17, 2004)	219.6	270.2	539	
Storm (Oct. 12, 1983)	2,957.3	5,741.4	8,783	Storm (Oct. 1, 2004)	421.2	748.7	1,508	
Storm (Oct. 14, 1983)	2,459.0	5,643.0	9,874	Storm (May 9, 2005)	3,198.4	3,776.3	6,454	
Storm (Jun. 2, 1984)	3,390.7	7,144.7	11,642	Storm (Sep. 10, 2005)	598.5	923.6	1,962	
Storm (Nov. 18, 1984)	4,739.7	5,090.5	10,639	Storm (Dec. 4, 2005)	376.4	709.4	1,434	
NELSON (Aug. 22, 1985)	4,896.8	8,270.4	14,228	Storm (Dec. 11, 2005)	1,500.8	3,511.4	5,612	
ALEX (Jul. 27, 1987)	2,188.9	3,073.5	5,500	Storm (Jun. 6, 2006)	881.2	1,444.3	2,937	
ABE (Aug. 30, 1990)	4,298.7	8,411.9	10,829	Storm (Jun. 15, 2007)	1,112.8	2,120.7	4,039	
Storm (Sep. 1, 1990)	570.9	2,743.9	5,100	Storm (Sep. 4, 2007)	1,807.4	2,499.0	4,706	
Storm (Sep. 2, 1990)	4,342.3	5,782.3	11,032	Storm (Oct. 10, 2008)	2,299.4	3,656.0	5,911	
Statistics	$Q_s/Q_{ m obs}$		$Q_q/Q_{\rm obs}$	Statistics	$Q_s/$	$Q_{\rm obs}$	$Q_q/Q_{\rm obs}$	
Maximum	0.478		Ó.777	Maximum	0.4	496	Ó.626	
Minimum	0.112		0.478	Minimum	0.2	262	0.420	
Mean	0.351		0.587	Mean	0.3	360	0.525	
Standard deviation	0.086		0.073	Standard deviation	0.0)78	0.054	
Coefficient of variation	0.245		0.124	Coefficient of Variation	0.2	216	0.104	

Note: Bold values represent simulation results of events whose CE values are larger than or equal to 0.8. Q_q = total discharge of simulated quick flow; Q_s = total discharge of simulated slow flow; and Q_{obs} = total discharge of streamflow observation.

where the Q_q symbol represents total discharge of simulated quick flow; the Q_s symbol denotes total discharge of simulated slow flow, and the Q_{obs} symbol is total discharge of streamflow observation. Unlike the regression results of time to peak and peak discharge, the smallest R^2 value is 0.80, and the other three values are even more than 0.9. These regression results provide sufficient evidence to reveal linear correlations of total discharges between component hydrographs and streamflow observations based on coefficients of variation of small ranges and R^2 results of high values. The linear ratios of total discharges between runoff components and streamflow in the early period are close to those in the later periods. The differences between the two time periods are small.

On the basis of comparison of the results presented, this study concludes the peak discharge of a quick runoff usually exceeds that of a slow runoff, whereas the peak discharge of a quick runoff approaches that of a total runoff for the same event. In a typhoon or large storm, the total discharges of quick runoffs are markedly larger than those of slow runoffs and slightly smaller than those of streamflow hydrographs in rainfall-runoff generations. Furthermore, the ratio percentages for total discharges between hydrograph components and streamflow observations are increasingly linear correlations.

Conclusions

This study uses a model of three serial reservoirs with one parallel reservoir and eight significant parameters to evaluate the shape characteristics of runoff components in a watershed outlet in Taiwan. The parameter limitations of hydrological modeling demonstrate the prerequisite of conforming to a physical phenomenon in a hydrological cycle. These limitations can offer effective assistance when observing runoff components of large events in the streamflows of a watershed outlet. The results of this study can help researchers determine the shape characteristics of component hydrographs using only rainfall and streamflow observations, without complex model simulation. In addition, the shape characteristics of other watersheds in Taiwan can also be employed to rapidly draw their component hydrographs. Evaluation results resulting from the data in this and other watersheds can be combined to improve watershed management in Taiwan.

The calibration and verification results for model efficiency confirm that the proposed model is suitable for evaluating runoff components relating to rainfall-streamflow observations in this watershed and can fit data from basins in other parts of Taiwan. From the calibration results of event data in early and later periods, time to peak of a quick runoff is the same as that of a streamflow, and time to peak of a slow runoff is later than that of a quick runoff. In the same typhoon or large storm, the peak discharge of a quick runoff usually exceeds that of a slow runoff, whereas the peak discharge of a quick runoff approaches that of the streamflow. When considering a typhoon or a large storm, the total discharge of a quick runoff is obviously larger than that of a slow runoff and slightly smaller than the streamflow. A quick/surface runoff creates a sharp point in a span after rainfall and terminates within a short period, whereas a slow flow gradually stops after a long period. The shape of a quick runoff hydrograph is more pointed and shifts forward compared with a slow flow. The quick and slow runoffs regenerated from the proposed model reveal satisfactory characteristics of runoff components.

This study also shows that times to peak for both hydrograph components may correspond to the peak time of a hyerograph; peak discharges of hydrograph components may be linearly related to those in the observed streamflow; and relationships of total discharges also have direct ratios between hydrograph components and streamflow observations. The correlation results, time, and flow characteristics of hydrograph components related to rainfall and streamflow observation create small effects on the early and later periods. Using the proposed procedure, this study uses three shape characteristics of component hydrographs to determine rapidly the shapes of simple hydrographs. For example, the triangle hydrographs of runoff components are only based on recordings of rainfall and streamflow, without complex model simulation.

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