Groundwater

Technical Commentary/

Why Hydraulic Tomography Works?

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Head measurements at a single observation well during a cross-hole pumping test carry a great amount of information about aquifer heterogeneity other than the average property of the aquifer as implied in Theis analysis of aquifer test. In this commentary, we use simple examples and a probabilistic reasoning approach based on Darcy's law to unravel this information, buried in the results of quantitative stochastic analyses of flow in heterogeneous aquifers (Bakr et al. 1978; Dagan 1985, 1989) and vadose zones (Yeh et al. 1985a, 1985b, 1985c; Yeh and Zhang 1996). We subsequently use this information to elucidate the principles of hydraulic tomography (HT), sequential pumping tests, or multi-well interference tests (see Yeh and Liu 2000; Illman et al. 2009; Brauchler et al. 2011; Cardiff and Barrash 2011).

Consider a pumping test in a one-dimensional heterogeneous confined aquifer (i.e., a horizontal soil column) which contains a pumping and an observation port. Ends of the aquifer are held at the same prescribed constant head, flow is at steady state, and the pumping rate, Q, is known. We now ask what the pumping rate and the drawdown at the observation port tell us about the spatial variation of the aquifer hydraulic conductivity (K).

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Received May 2013, accepted September 2013.

doi: 10.1111/gwat.12129

To answer this question qualitatively, we consider an aquifer which has 10 elements, and a pumping port at its center (Figure 1a). If the K values of the 10 elements are unknown, one can guess an infinite number of possible K fields. These guessed K fields lead to an infinite number of possible head distributions between the pumping port and the two boundaries that satisfy the given Q, and the boundary conditions. That is, if aquifer properties are not specified, the forward model has nonunique solutions, and it is ill-posed according to Hadamard's (1902) definition.

Conceptually, these nonunique head solutions are bounded by an upper and a lower limits and have a unique mean head field for the given layout and boundary conditions (Figure 1a). This mean head denotes the average of all possible head distributions bounded by the two limits. It is called unconditional mean head since it is not constrained by any observed head. The upper bound is approximated by our guess of the head (h_5) at the pumping location x_5, which must be slightly smaller than the head at x_0 (h_0) because of the pumping. Similarly, the fact that the lower bound head at x_1 (h_1) must be slightly larger than a guessed head at x_5 leads to the lower bound. On the basis of these upper and lower head bounds (in turn, the minimum and maximum gradients), and the known Q (assuming that a half of the Q is from the right and the other half from the left), one can qualitatively determine the upper and the lower limits of K values for the 10 elements. Subsequently, one can obtain an unconditional K value (K_u) from the unconditional mean head distribution and Q.

Suppose the drawdown (or head) at the observation port at x_3 (h_3) is the only head measurement and the pumping rate is known. If this head is higher than the unconditional mean head based on K_u, one would guess that the true average gradient from both boundaries toward pumping port is smaller than the unconditional mean head gradient. Accordingly, the harmonic average of the true K values for the five blocks on both sides of the aquifer (Freeze and Cherry 1979) should be greater than K_u and vice versa. Therefore, statistically the observed head is positively correlated with the K_u on each side of the pumping port.

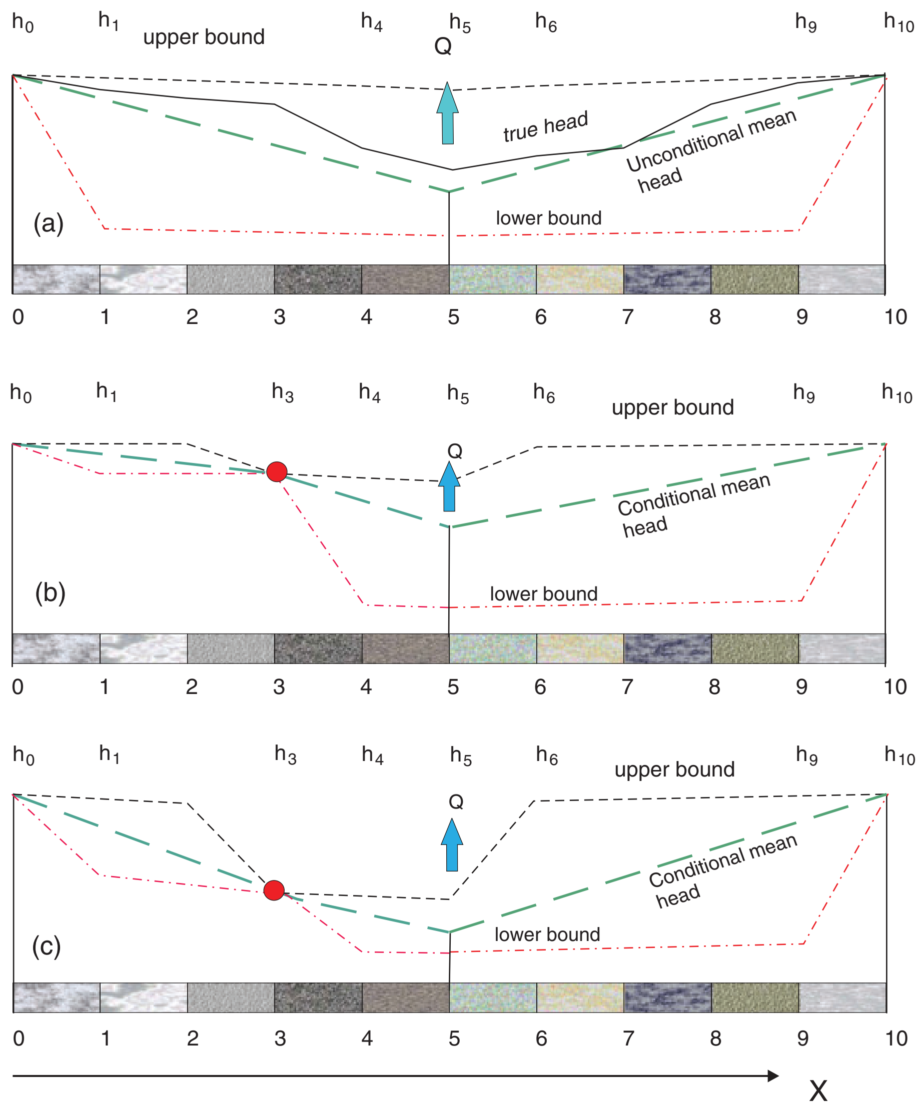

Figure 1. Illustrations for explaining the relationship between an observed head and aquifer heterogeneity. The red circle indicates the observed head, and the head values are arbitrarily scaled.

Next, we investigate the relationship between h_3 and possible K values of the five blocks from the left boundary to the pumping port. First of all, all possible head distributions in this situation must agree with h_3 at the observation port (i.e., conditioning). Then, the previous reasoning approach can yield their upper and the lower bounds and a conditional mean head distribution. There can be the following two possibilities for the conditional mean head distribution: Case 1, the observed head is higher than the unconditional mean head and Case 2, the observed head is lower. The conditional mean head distributions (green long dashed line) and their upper and lower bounds (black and red short dashed lines, respectively) for Cases 1 and 2 are illustrated in Figure 1b and 1c, respectively.

In Case 1, the gradient of the conditional mean head between the left boundary and the observation port (i.e., upstream region of the observation port) in Figure 1b is flatter than the gradient of the corresponding unconditional mean head in Figure 1a. The conditional mean head gradient between the observation and the pumping ports (i.e., downstream region from the observation port) is steeper than the corresponding unconditional mean head gradient. Since flow is steady, the flux along the flow path must be constant. The conditional effective K or K_c (i.e., the harmonic average of K values of the three blocks in the upstream region) therefore is greater than the K_c of the two blocks in the downstream region. In Case 2 (Figure 1c), the relationships between the K_c upstream and downstream regions are opposite to those of Case 1.

On the basis of the above discussions, we therefore conclude that the head at the observation port during the pumping test can reveal relative magnitudes of K_c of the upstream and the downstream of the observation regions as well as the opposite region (i.e., the region between the pumping port and the right boundary). Namely, the head is positively correlated with the upstream K_c and negatively correlated with the downstream one. It is also positively correlated with the K_c of the opposite region. Similar results were reported by Wu et al. (2005), Mao et al. (2011, 2013), and Sun et al. (2013), in which they employed a first-order stochastic analysis based on sensitivity to derive such correlations for more complex radial flow fields in two and three dimensional (2D and 3D) aquifers.

Such a correlation implies that with the pumping location fixed, a head at a new observation location may reflect different relative K_c values of the upstream, downstream, and opposite regions from those based on the head at previous observation location. That is, the new head carries nonredundant (although inexact) information about individual K within each region. A joint interpretation of heads at many locations thereby reduces the uncertainty of the value of each K. Likewise, heads at the same observation location due to pumping at different locations (a change of flow field) could carry nonredundant information. Jointly interpreting these heads thus could yield a high-resolution map of the spatial K distribution. This is exactly the principle behind HT.

To demonstrate this principle quantitatively, an aquifer with a similar setup as in Figure 1 was used. The 10 true K values were generated with a mean of 1.24 m/d, a variance equal to 0.69 m^2/d^2, and 1-m correlation scale, and they are shown as dark blue bars in Figure 2a, 2b, and 2c. Three steady heads at $x = 3$ m corresponding to pumping test at $x = 5$, 7, and 9 m were then simulated with a discharge Q equal to 0.2 m^3/d and the two boundary heads equal to 100 m. These three heads were then used to demonstrate the effectiveness of HT for estimating the 10 K values. The joint interpretation for K_s was carried out using the successive linear estimator (SLE) (Yeh et al. 1996; Yeh and Liu 2000; Zhu and Yeh 2005; Xiang et al. 2009). Specifically, the estimates were obtained via

$$K_i^r = K_i^{r-1} + \alpha_{i3}^{r-1}\left(h_{35}^{r-1} - H_{35}\right) \text{ for pumping at } x = 5 \text{ m} \tag{1}$$

$$K_i^r = K_i^{r-1} + \alpha_{i3}^{r-1}\left(h_{35}^{r-1} - H_{35}\right) + \beta_{i3}^{r-1}\left(h_{37}^{r-1} - H_{37}\right) \text{ for pumping at } x = 5 \text{ and } 7 \text{ m} \tag{2}$$

$$K_i^r = K_i^{r-1} + \alpha_{i3}^{r-1}\left(h_{35}^{r-1} - H_{35}\right) + \beta_{i3}^{r-1}\left(h_{37}^{r-1} - H_{37}\right) + \lambda_{i3}^{r-1}\left(h_{39}^{r-1} - H_{39}\right) \text{ for pumping at } x = 5, 7, \text{ and } 9 \text{ m} \tag{3}$$

where K_i^r is the estimated K at element $i(i = 1, \ldots, 10)$ and at the end of the rth iteration, and when $r = 0$, K *is* the unconditional mean value. The weights, α_{i3}^{r-1}, β_{i3}^{r-1} and λ_{i3}^{r-1}, were calculated from the correlation between the K value of element i at iteration $r - 1$ and the difference between simulated head (h_{35}^{r-1}, $h_{37}^{r-1} h_{39}^{r-1}$) and observed head ($H_{35}$, H_{37}, and H_{39}) at location 3 due to pumping at locations 5, 7, and 9, respectively. Iteration is required because of the nonlinear relationship between head and K in Darcy's law and linear nature of the estimator (Equations 1, 2, and 3). The simulated heads (h_{35}^{r-1}, $h_{37}^{r-1} h_{39}^{r-1}$) are the head evaluated with Darcy's law using K values at iteration $r - 1$ and with the corresponding pumping operations.

Figure 2a shows the estimated K of the 10 elements as light blue bars; it also shows that the observed head (solid green line) at $x = 3$m is lower than the unconditional mean head (dash green line) owing to pumping at $x = 5$m. On the basis of the estimated K values, K_c of the five elements on the left-hand side of the pumping port is 0.85 m/d and that on the right-hand side is 0.88 m/d. As discussed previously, both are smaller than the unconditional mean K_u (1.00 m/d, our initial guess value). Furthermore, K_c of the three elements upstream from $x = 3$m is 0.81m/d, and it is lower than K_c of the two elements downstream (0.92 m/d). That is, a single observed steady head during a single pumping test can reveal the spatial pattern of K_c.

When the head at the same observation port due to pumping at $x = 7$ m was used in addition to that in Figure 2a for estimation, the estimated K field is shown in Figure 2b. The estimated K field using heads observed at $x = 3$m due to sequential pumping tests at $x = 5$, 7, and 9 m are plotted in Figure 2c. Noticeably, even though the head at the pumping port is not measured, adding more pumping tests at locations to the right of the observation port improves the K estimates on the right. The improvement is attributed to the fact that as the pump moves to the right, the number of Ks comprising the K_c of the downstream region increases and that of the opposite region decreases. As a consequence, the correlation pattern between the observed head and Ks within the two region changes, while the members of the K_c for the upstream region and their correlation pattern remain the same. Thus, there is no improvement on the K pattern of that region although the overall values have changed.

Furthermore, the change in the pattern of the K estimates from Figure 2a to Figure 2b shows that the improvements are not limited only to the area bounded by the observation and the pumping locations. This is expected from the correlation pattern as discussed above, and it is consistent with the results reported by Sun et al. (2013) for 2D radial flow problems.

In conclusion, a head measurement during a cross-hole pumping test contains important information about aquifer heterogeneity. HT recognizes this information and collects head data that contain nonredundant information about aquifer heterogeneity to improve the estimation of parameters. This note echoes with our call to "change the way we collect and analyze data for characterizing aquifers" (Yeh and Lee 2007).

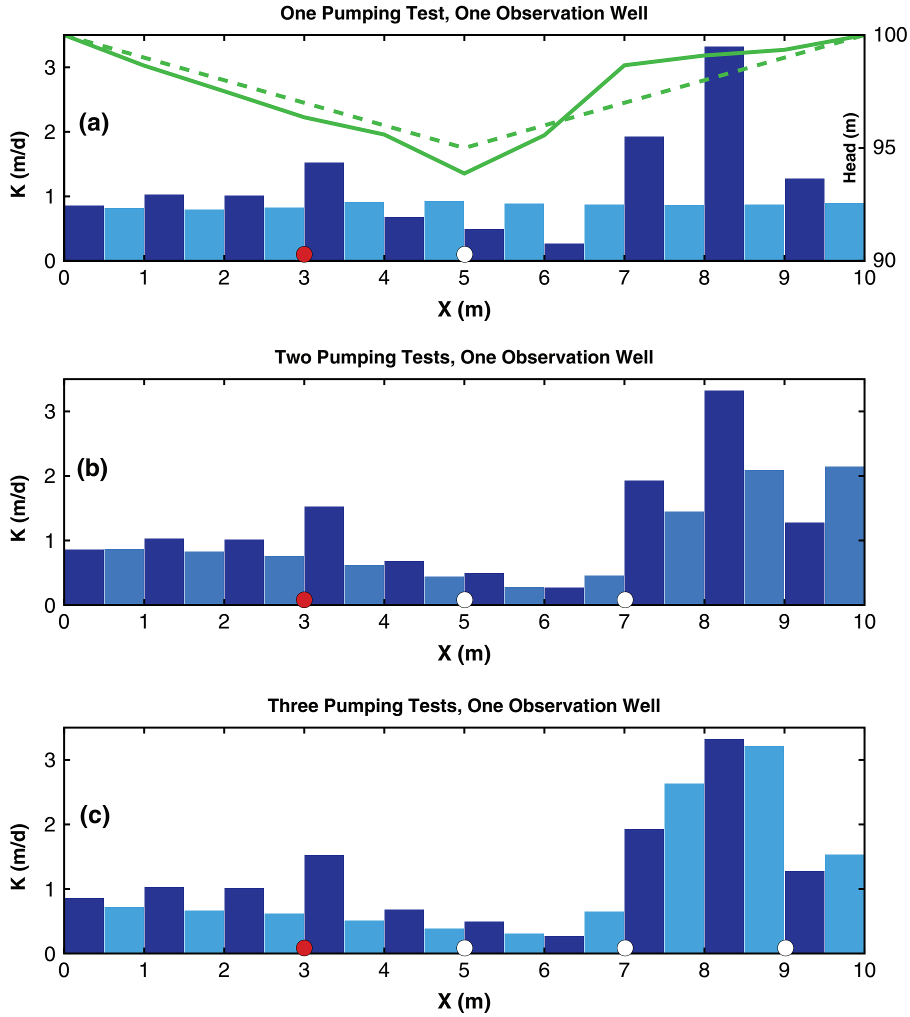

Figure 2. Progressive improvements of *K* estimates using observed heads at the same monitoring port while pumping at different locations. The true *K* values (dark blue bars) and the estimates (light blue bars) are plotted next to each other in each of the 10 elements. Solid line and dashed line in (a) are the true and unconditional mean head distributions in the aquifer, respectively. The red circle is the location of the monitoring port, and the white circles denote the pumping port.

Acknowledgments

This research is supported by the NSF grant EAR 1014594. Tian-Chyi J. Yeh also acknowledges support of the oversea outstanding scholar award of Department of Education, China, through Jilin University, China. Deqiang Mao and Yuanyuan Zha acknowledge the support from the "China Scholarship Council."

References

Bakr, A.A., L.W. Gelhar, A.L. Gutjahr, and J.R. MacMillan. 1978. Stochastic analysis of spatial variability in subsurface flows: 1. Comparison of one- and three-dimensional flows. *Water Resources Research* 14, no. 2: 263–271. DOI:10.1029/WR014i002p00263

Brauchler, R., R. Hu, P. Dietrich, and M. Sauter. 2011. A field assessment of high-resolution aquifer characterization based on hydraulic travel time and hydraulic attenuation tomography. *Water Resources Research* 47: W03503. DOI:10.1029/2010WR009635

Cardiff, M., and W. Barrash. 2011. 3D transient hydraulic tomography in unconfined aquifers with fast drainage response. *Water Resources Research* 47: W12518. DOI:10.1029/2010WR010367

Dagan, G. 1989. *Flow and Transport in Porous Formations*, 244. Berlin, Germany: Springer-Verlag.

Dagan, G. 1985. Stochastic modeling of groundwater flow by unconditional and conditional probabilities:' The inverse problem. *Water Resources Research* 21, no. 1: 65–72.

Freeze, R.A., and J.A. Cherry. 1979. *Groundwater*. Englewood Cliffs, New Jersey: Prentice-Hall, Inc.

Hadamard, J. 1902. Sur les problèmes aux dérivés partielles et leur signification physique. *Princeton University Bulletin* 13: 49–52.

Illman, W.A., X. Liu, S. Takeuchi, T.-C.J. Yeh, K. Ando, and H. Saegusa. 2009. Hydraulic tomography in fractured granite: Mizunami underground research site, Japan. *Water Resources Research* 45: W01406. DOI:10.1029/2007WR006715

Mao, D., T.-C. J. Yeh, L. Wan, C.-H. Lee, K.-C. Hsu, J.-C. Wen, and W. Lu 2013. Cross-correlation analysis and information content of observed heads during pumping in unconfined aquifers. *Water Resources Research* 49. DOI:10.1002/wrcr.20066

Mao, D., L. Wan, T.-C.J. Yeh, C.-H. Lee, K.-C. Hsu, J.-C. Wen, and W. Lu. 2011. A revisit of drawdown behavior during pumping in unconfined aquifers. *Water Resources Research* 47: W05502. DOI:10.1029/2010WR009326

Sun, R., T.-C.J. Yeh, D. Mao, M. Jin, W. Lu, and Y. Hao. 2013. A temporal sampling strategy for transient hydraulic tomography analysis. *Water Resources Research* 49: 3881–3896. DOI:10.1002/wrcr.20337

Wu, C.-M., T.-C.J. Yeh, J. Zhu, T.-H. Lee, N.-S. Hsu, C.-H. Chen, and A.F. Sancho. 2005. Traditional analysis of aquifer tests: Comparing apples to oranges. *Water Resources Research* 41, no. 9: W09402.

Xiang, J., T.-C.J. Yeh, C.-H. Lee, K.-C. Hsu, and J.-C. Wen. 2009. A simultaneous successive linear estimator and a guide for hydraulic tomography analysis. *Water Resources Research* 45, no. 2: W02432.

Yeh, T.-C.J., and C.-H. Lee. 2007. Time to change the way we collect and analyze data for aquifer characterization. *Ground Water* 45, no. 2: 116–118.

Yeh, T.-C.J., and S. Liu. 2000. Hydraulic tomography: Development of a new aquifer test method. *Water Resources Research* 36, no. 8: 2095–2105.

Yeh, T.-C.J., M. Jin, and S. Hanna. 1996. An iterative stochastic inverse method: conditional effective transmissivity and hydraulic head fields. *Water Resources Research* 32, no. 1: 85–92.

Yeh, T.-C.J., and J. Zhang. 1996. A geostatistical inverse method for variably saturated flow in the vadose zone. *Water Resources Research* 32, no. 9: 2757–2766.

Yeh, T.-C.J., L.W. Gelhar, and A.L. Gutjahr. 1985a. Stochastic analysis of unsaturated flow in heterogeneous soils: 1, statistically isotropic media. *Water Resources Research* 21, no. 4: 447–456.

Yeh, T.-C.J., L.W. Gelhar, and A.L. Gutjahr. 1985b. Stochastic analysis of unsaturated flow in heterogeneous soils: 2, statistically anisotropic media. *Water Resources Research* 21, no. 4: 457–464.

Yeh, T.-C.J., L.W. Gelhar, and A.L. Gutjahr. 1985c. Stochastic analysis of unsaturated flow in heterogeneous soils: 3, observations and applications. *Water Resources Research* 21, no. 4: 465–471.

Zhu, J., and T.-C.J. Yeh. 2005. Characterization of aquifer heterogeneity using transient hydraulic tomography. *Water Resources Research* 41: W07028.