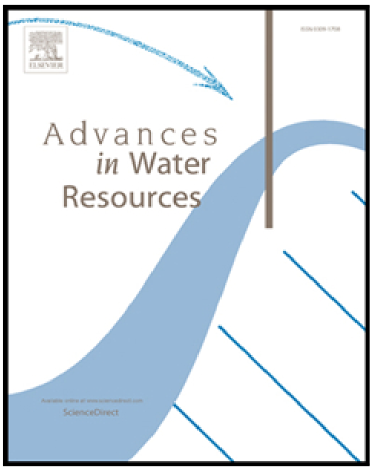

Quasi-Steady State Conditions in Heterogeneous Aquifers during Pumping Tests

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Highlights

- ¶ Definitions of approximate quasi-steady flow due to pumping in aquifers
- ¶ Influences of heterogeneity on approximate quasi-steady
- ¶ Temporal evolution of information content in head observations
- ¶ Field data demonstrating lack of quasi-steady state

Quasi-Steady State Conditions in Heterogeneous Aquifers during Pumping Tests

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ABSTRACT

Classical Thiemôs well hydraulic theory, other aquifer test analyses, and flow modeling efforts often assume the existence of ñquasi-steadyò state conditions. That is, while drawdowns due to pumping continue to grow, the hydraulic gradient in the vicinity of the pumping well does not change significantly. These conditions have built upon two-dimensional and equivalent homogeneous conceptual models, but few field data have been available to affirm the existence of these conditions. Moreover, effects of heterogeneity and three-dimensional flow on this quasi-steady state concept have not been thoroughly investigated and discussed before. In this study, we first present a quantitative definition of quasi-steady state (or steady-shape conditions) and steady state conditions based on the analytical solution of two- or three-dimensional flow induced by pumping in unbounded, homogeneous aquifers. Afterward, we use a stochastic analysis to investigate the influence of heterogeneity on the quasi-steady state concept in heterogeneous aquifers. The results of the analysis indicate that the time to reach an approximate quasi-steady state in a heterogeneous aquifer could be quite different from that estimated based on a homogeneous model. We find that heterogeneity of aquifer properties, especially hydraulic conductivity, impedes the development of the quasi-steady state condition before the flow reaching steady state. Finally, 280 drawdown-time data from the hydraulic tomographic survey conducted at a field site corroborate our finding that the quasi-steady state condition likely would not take place in heterogeneous aquifers unless pumping tests last a long period.

Research Significance

1) Approximate quasi-steady and steady state conditions are defined for two- or three-dimensional flow induced by pumping in unbounded, equivalent homogeneous aquifers. 2) Analysis demonstrates effects of boundary condition, well screen interval, and heterogeneity of parameters on the existence of the quasi-steady, and validity of approximate quasi-steady concept. 3) Temporal evaluation of information content about heterogeneity in head observations are analyzed in heterogeneous aquifer. 4) 280 observed drawdown-time data corroborate the stochastic analysis that quasi-steady is difficult to reach in highly heterogeneous aquifers.

Keywords:

Ergodicity; quasi-steady; stochastic analysis; pumping

1. INTRODUCTION

Multi-scale heterogeneity of aquifers is the rule rather than the exception. Nevertheless, widely-accepted analyses of cross-hole pumping tests adopt an equivalent homogeneous conceptual model [39] to homogenize aquifer heterogeneity. Using a stochastic analysis, Wu et al. [36] showed that the governing equation for the equivalent homogeneous model is an ensemble mean equation, embedding with effective transmissivity and storage coefficient. As such, it represents the physical principle governing the average flow over many possible realizations (i.e., an ensemble) of flow fields under the same stress, and it predicts ensemble mean hydraulic head fields [39]. As a result, as one applies this model to a real-world aquifer, one inevitably invokes the ergodicity assumption (i.e., the ensemble average is equivalent to the spatial average [26]). Specifically, the predicted mean heads at a given radial distance from the pumping well will be equivalent only to the averages of heads at different locations at the same radial distance in a heterogeneous aquifer. Wu et al. [36] subsequently advocated that using observed drawdown-time data at one observation well in an equivalent homogeneous model to estimate aquifer properties is tantamount to comparing apples and oranges. They further showed that the estimated aquifer properties from Theis solution [32] or Cooper and Jacobôs approach [7] using one well hydrograph are ambiguously averaged properties over the cone of depression. More specifically, rather than the average values of aquifer properties over the cone of depression, the transmissivity estimate based on late time drawdown data is heavily influenced by the heterogeneity near the pumping well and the observation well, and the storage coefficient estimate is mainly related to the heterogeneity between the pumping well and the observations.

Results of analysis of data from field experiments in [14,29,35] corroborated the findings by Wu et al. [36]. They further suggested that the estimated parameters using an equivalent homogeneous model are scenario-dependent: they vary with duration of the pumping and the location of the pumping well. Yeh et al. [39] and Yeh and Lee [40] pointed out that the non-intrinsic natures of these estimates mainly arise from our ignorance of the ergodicity assumption behind the equivalent homogeneous models. That is, the flow itself must sample sufficient heterogeneity in the aquifer such that the ensemble mean equation is applicable. In addition to insufficient data, the basic assumption (i.e., the form of the equivalent model) may also involve uncertainty. For instance, the selection of single-porosity or dual-porosity model may have significant impact on the equivalent parameter (especially storage coefficient) as well as the scale to reach ergodicity condition [23]. For this reason, Yeh and Lee [40] emphasized the necessity of detailed characterizations of the spatial distributions of hydraulic properties in order to minimize these problems.

Similar to the homogeneity assumption, quasi-steady state assumption has been widely accepted and employed in the analysis of aquifer tests. For example, the well-known Thiem equation [31] assumes the existence of an effective area of inference during a pumping test and suggests the use of steady state solution to estimate hydraulic conductivity. It is also common to assume the establishment of quasi-steady flow near the pumping well during tracer tests, so that the solute transport can be studied analytically or numerically under steady velocity field [e.g., 17, 24]. Heath and Trainer [12] stated that if quasi-steady state conditions (called steady-shape conditions) apply to near the well, Thiem equation is applicable. Butler [6] pointed out that steady-shape conditions are reached when t=100$r^2S/(4T)$, where r is the distance between the pumping well and observation well, S is the storage coefficient, and T is the transmissivity. More recently, Bohling et al. [4-5] and Hu et al. [13] championed the robustness of this assumption for cutting down computational costs in analyzing hydraulic tomography (HT). The importance of steady-shape conditions in practice was reemphasized by Health [11]. For practical modeling applications, Domenico and Schwartz [10] proposed an aquifer system time constant for aquifer. They claimed that if the time at which we wish to observe the system is much larger than the time constant, the system will appear to be at steady state, and the system can be simulated using a steady-state model. Based on this suggestion, Anderson et al. [2] discussed one groundwater modelersô fundamental decision---where a transient model is needed. They stated that since steady-state models are much easier to operate than transient models, the formers are typically preferred provided they adequately address the modeling objective.

By assuming existence of quasi-steady conditions in a statistically homogeneous and horizontally isotropic aquifer, Neuman et al. [21] proposed a graph method to estimate the geometric mean, integral scale and variance of the log transmissivity field on the basis of quasi-steady data when a randomly heterogeneous, two-dimensional aquifer is pumped at a constant rate. Using numerical experiments, they showed that the mean and integral scale can be reasonably recovered if there were sufficient observations, but it was difficult to obtain accurate variance value. Neuman et al. [22] showed the existence of quasi-steady regime in heterogeneous aquifer with numerical experiment and field data. Nevertheless, Vasco and Karasaki [34] argued that in heterogeneous media, the onset of quasi-steady conditions might be delayed by the presence of low-conductivity regions, which fail to equilibrate with the surrounding medium.

The accuracy or validity of these applications of quasi-steady state conditions, however, are difficult to assess because of the following reasons: 1) Aquifers are inherently heterogeneous and flow is always three-dimensional. The number of wells in field experiments is limited and the wells are not fully penetrating the entire thickness of the aquifer as required by Theis solution. As a consequence, few field data have offered convincing evidence of the existence of quasi-steady state conditions. 2) The inverse solution for ill-defined problems (i.e., lack of the necessary conditions, see Mao et al. [20] and Yeh et al. [38-39]) always involve uncertainty. 3) The choice of the equivalent homogeneous model (e.g., single-porosity or dual-porosity model) may also have impact on the occurrence of the steady-shape condition [23]. The robustness of application of quasi-steady state conditions to an inverse modeling problem thus is still in question. Here, we focus on the first issue to discuss the validity of steady-shape condition.

In this study, we first offer a quantitative definition of quasi-steady state condition in unbounded homogeneous aquifers. Afterwards, the validity of quasi-steady condition in bounded heterogeneous aquifers is analyzed using the stochastic concept and approach. The temporal evolution of cross-correlations between parameters and the observed drawdown is considered subsequently. At last, a large number of observed drawdown-time curves due to pumping in a field are examined. We then discuss implications of the results and present our conclusions.

2. QUASI-STEADY STATE IN EQUIVALENT HOMOGENEOUS AQUIFERS

2.1 Two-dimensional, Homogeneous Aquifers

Based on the equivalent homogeneous conceptual model, a quasi-steady (or steady-state shape) condition can be defined if the temporal changes of hydraulic gradients between all available observation wells are "sufficiently" small. In order to derive a quantitative definition, we will start from the governing equation of two-dimensional flow in homogeneous and isotropic confined aquifer and assume the aquifer is unbounded in all lateral directions. With these assumptions, an analytical solution for the drawdown at a radial distance *r* from a pumping well was reported by Theis [32],

$$s(r,t) = \frac{Q}{4\pi T} W(u) \tag{1}$$

where *s* is the drawdown (initial head minus head at time *t*), $W(u) = \int_u^{\infty} \frac{e^{-z}}{z} dz$ is the well function and $u = r^2 S/(4Tt)$, *S* is the storage coefficient, *t* is time, *T* is transmissivity, *Q* is the constant pumping rate.

According to this solution, the hydraulic gradient *g* along radius direction at (*r*, *t*) is:

$$g = -\frac{\partial s}{\partial r} = \frac{Q}{2\pi r T} \exp\left[-\frac{r^2 S}{4Tt}\right] = \frac{Q}{2\pi r T} \exp(-u) \tag{2}$$

At late time (i.e., small *u*), the gradient *g* at any location *r* will asymptotically approach the value of $g_{asym} = Q/(2\pi rT)$, which is independent of time. As *u* is smaller than 0.01, the relative difference between $g(t)$ and g_{asym} will be less than 1% according to the mathematical properties of the function exp(-*u*). Under this situation, we can say that a quasi-steady of the cone of depression will exist, if we accept this 1% as the criterion. This situation has been referred to as steady-shape conditions.

Another way to look at this issue is to use the temporal derivative of drawdown (w),

$$w = \frac{\partial s}{\partial t} = \frac{Q}{4\pi Tt} e^{-\left(\frac{r^2 S_s}{4Tt}\right)} = \frac{Q}{4\pi Tt}\exp(-u) \quad (3)$$

Again, at late time (i.e., small u), the temporal change of head (w) at different locations (i.e., different r's) will asymptotically approach the value of $w_{asym} = Q/(4\pi tT)$, which is independent of location r. When u is smaller than 0.01, function $\exp(-u)$ will be greater than 0.99, which means that the relative difference of w values at different locations is less than 1%. This is tantamount to stating that head difference between any two different observation locations (a surrogate head gradient) is approximately constant in time with less than 1% relative error. This situation is commonly known as quasi-steady flow condition. As shown above, the quasi-steady condition is equivalent to the steady-shape condition. Note that a quasi-steady state does not imply approximate steady state, where the rate of change in the head is close to zero.

If we assume that the furthest observation well of the two wells is at r_m, in order to ensure all u's at the two locations are less than 0.01, t should be greater than $100 r_m^2 S/4T$, which is regarded as the time to reach quasi-steady condition (i.e., onset or kickoff time) for area $r \in (0, r_m]$ in the two-dimensional, unbounded, homogeneous aquifer. At this time, the water supplying rate (released water per unit time, with the same dimension as Q) from area $r \in (0, r_m]$ is:

$$2\pi \int_0^{r_m} S w(r,t)\, r\, dr = Q\left[1 - \exp\left(-\frac{r_m^2 S}{4Tt}\right)\right] \quad (4)$$

According to Eq. (4), in order to reach an approximate quasi-steady with less than 1% error, 99% of the pumped water must come from outside of the radius r_m. In this situation, the shape of the cone of depression for area r (0, r_m] does not change significantly and the quasi-steady of the depression approximately exist. As illustrated in Fig. 1(a), although drawdown *s* increases logarithmically with *t* and never attains steady state, the head gradient can be approximately regarded as solely a function of *r* within a circular quasi-steady state region, and the area of region (r^2) expands linearly with *t*. That is, the drawdown-log time lines at different *r* values will be parallel as illustrated in Fig. 1(a). This is the theoretical definition of the quasi-steady in a two-dimensional, homogeneous aquifer during a pumping test.

Next, we will examine the effects of three-dimensional flow on the definition of quasi-steady conditions. Notice that we have defined the quasi-steady state condition using either spatial or temporal derivatives of drawdown as a criterion. In the next section as well as the rest of this paper, we choose the temporal derivative of drawdown (*w*) as the criterion, rather than the spatial derivative or spatial gradient (*g*), although the latter is a more intuitive concept. Several reasons for this choice are: first, the spatial gradient (*g*) at a location cannot be accurately obtained since observation wells are often sparsely spaced. Second, to examine the quasi-steady, *N* drawdown-time curves usually require to calculate (N-1)N/2 pairs of head differences (see [4]). To the contrary, only time derivatives of these drawdowns are needed if time derivative of drawdown concept (*w*) is adopted.

2.2 Three-Dimensional, Homogeneous Aquifers

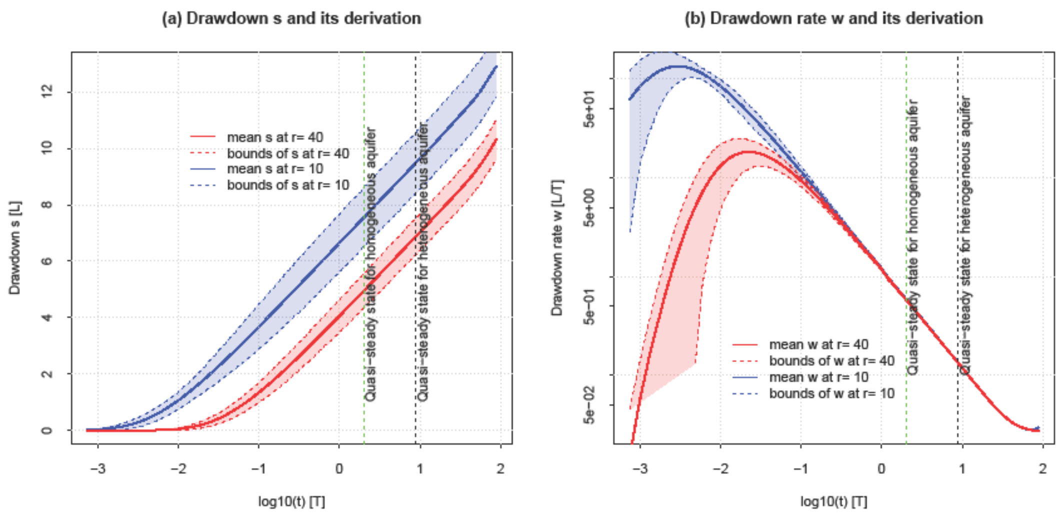
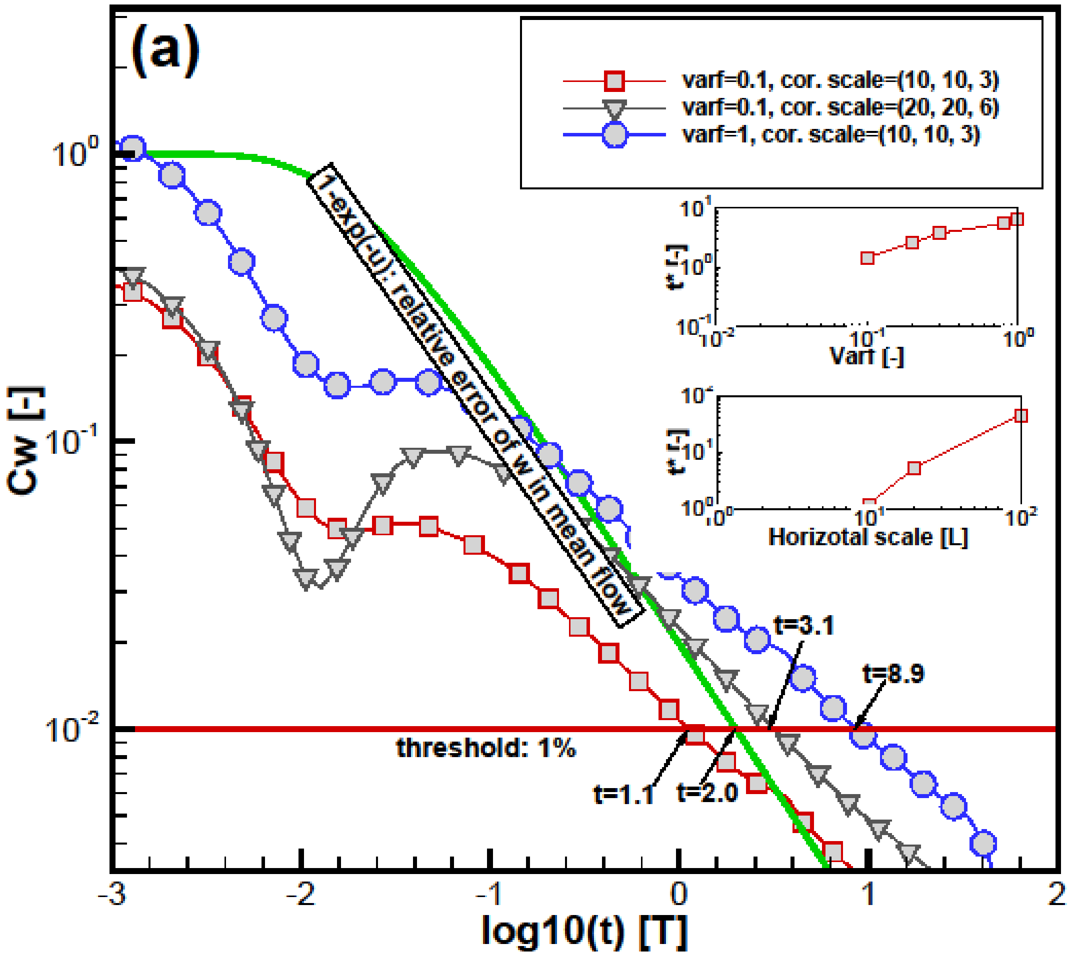


Fig. 6. Coefficient of variation C_w (defined by standard deviation of *w* normalized by *w*) versus time introduced by heterogeneity of (a) *f* or (b) *p*. The green line is 1-exp(-*u*) indicating the relative error of *w* in the mean flow according to Eq. (3). C_w should also be less than 1% based on the definition of steady shape condition in heterogeneous aquifer. The relationships between correlation scales or variances of *f* and *p* and the kickoff time *t** (when C_w<1%) are also displayed.


(a)
varf=0.1, cor. scale=(10, 10, 3)
varf=0.1, cor. scale=(20, 20, 6)
varf=1, cor. scale=(10, 10, 3)
1-exp(-u): relative error of w in mean flow
threshold: 1%
t=1.1
t=2.0
t=3.1
t=8.9
Cw [-]
log10(t) [T]
Varf [-]
Horizotal scale [L]
t* [-]

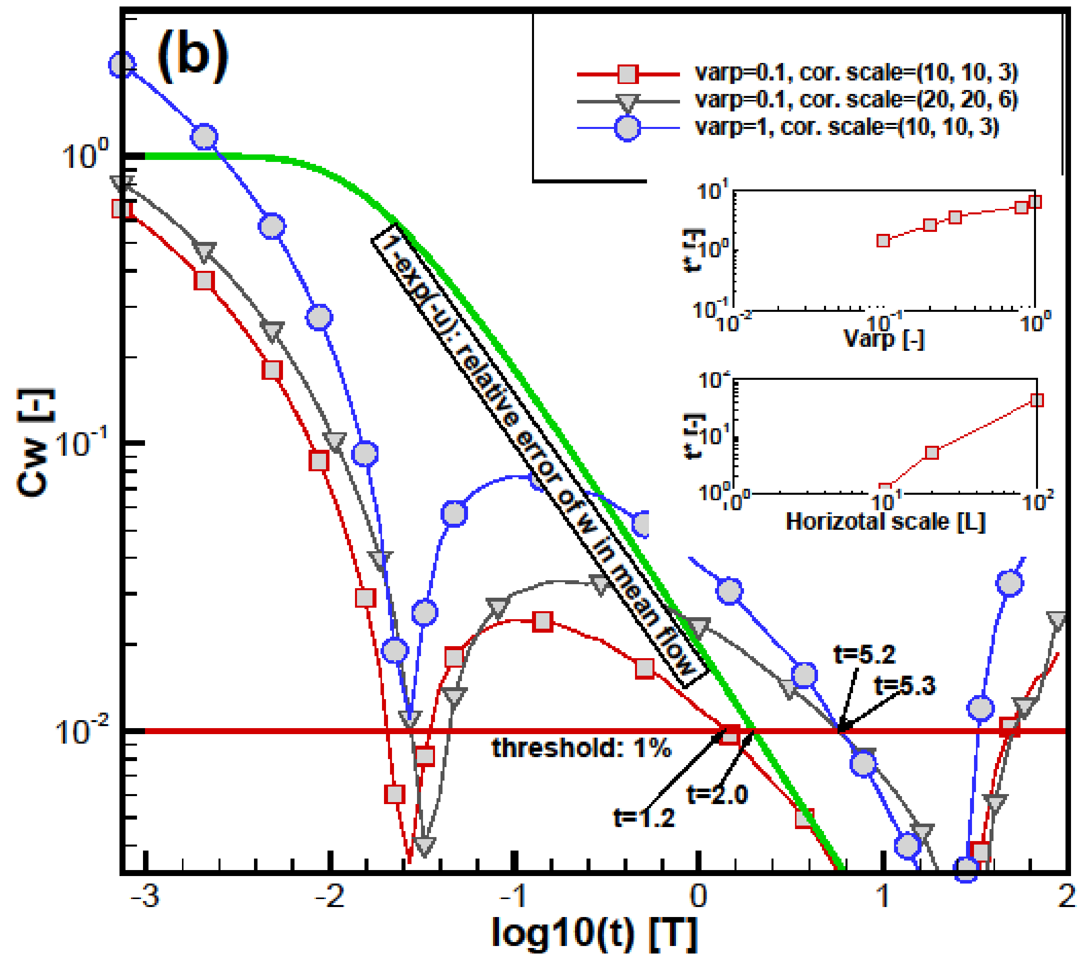


Fig. 7. The temporal evolutions of drawdown *s* and its derivative *w* at different observation locations (*r*=10 or 40). The shadow area indicating the bounds is calculated

by adding or subtracting standard derivation. Variance of p=1, correlation scales are 10, 10, and 3. Variance of f=0.

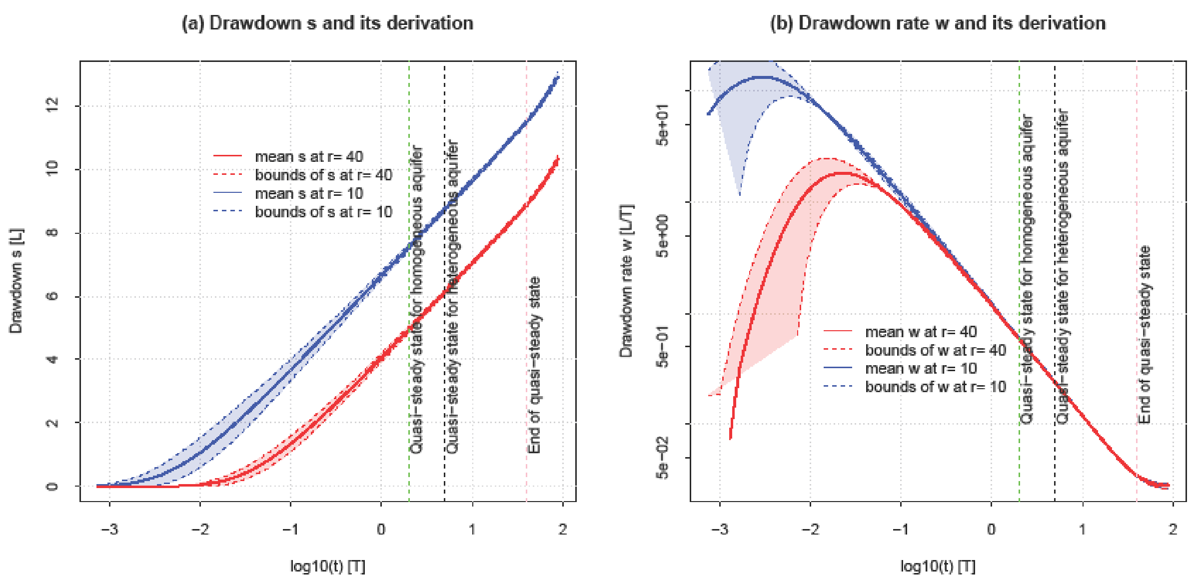
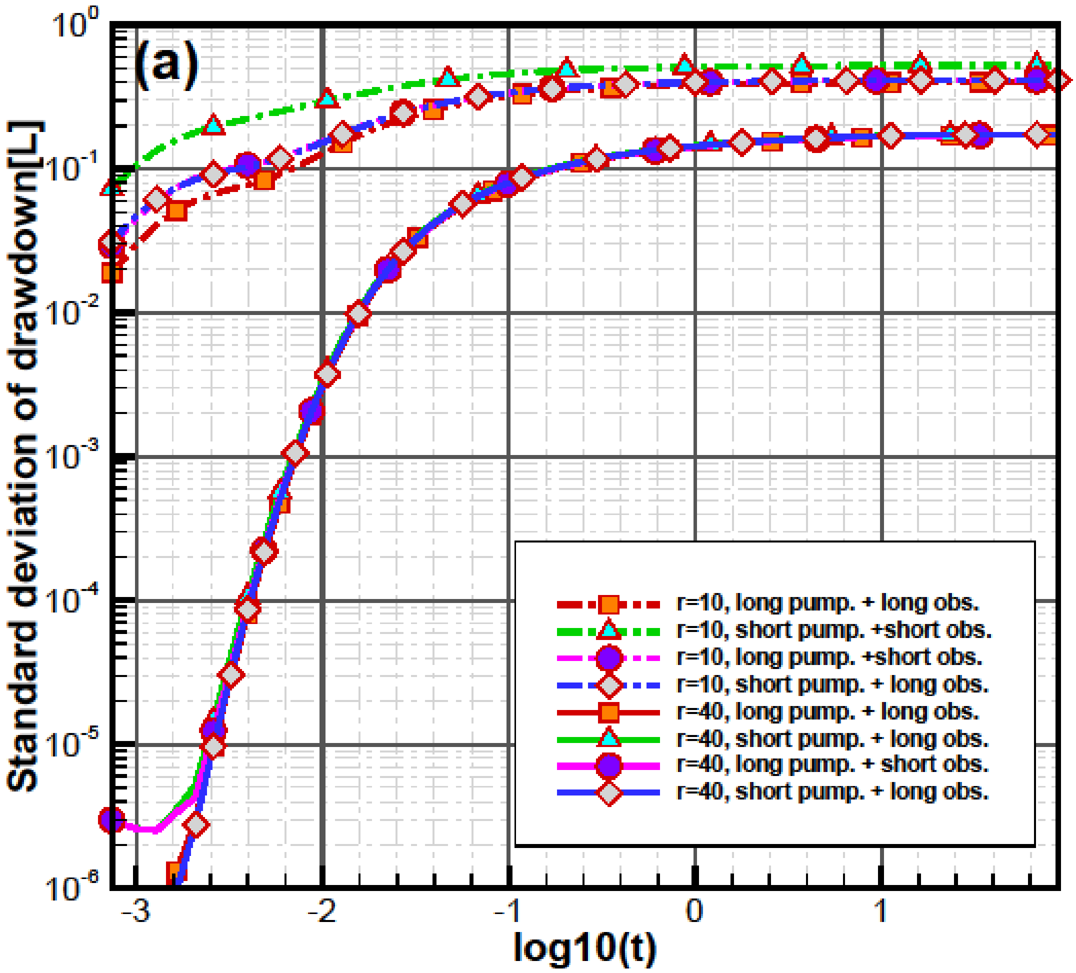


Fig. 8. The influences of well screen length on the variablity of drawndwon s (a) and its time derivative w (b) introduced by heterogeneity of K. The variablities are measured by the standard deviations of s or w. Variance of f=1, correlation scales are 10, 10, and 3. Variance of p=0.



(a)
Standard deviation of drawdown[L]
log10(t)
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
-3
-2
-1
0
1
r=10, long pump. + long obs.
r=10, short pump. +short obs.
r=10, long pump. +short obs.
r=10, short pump. + long obs.
r=40, long pump. + long obs.
r=40, short pump. + long obs.
r=40, long pump. + short obs.
r=40, short pump. + long obs.

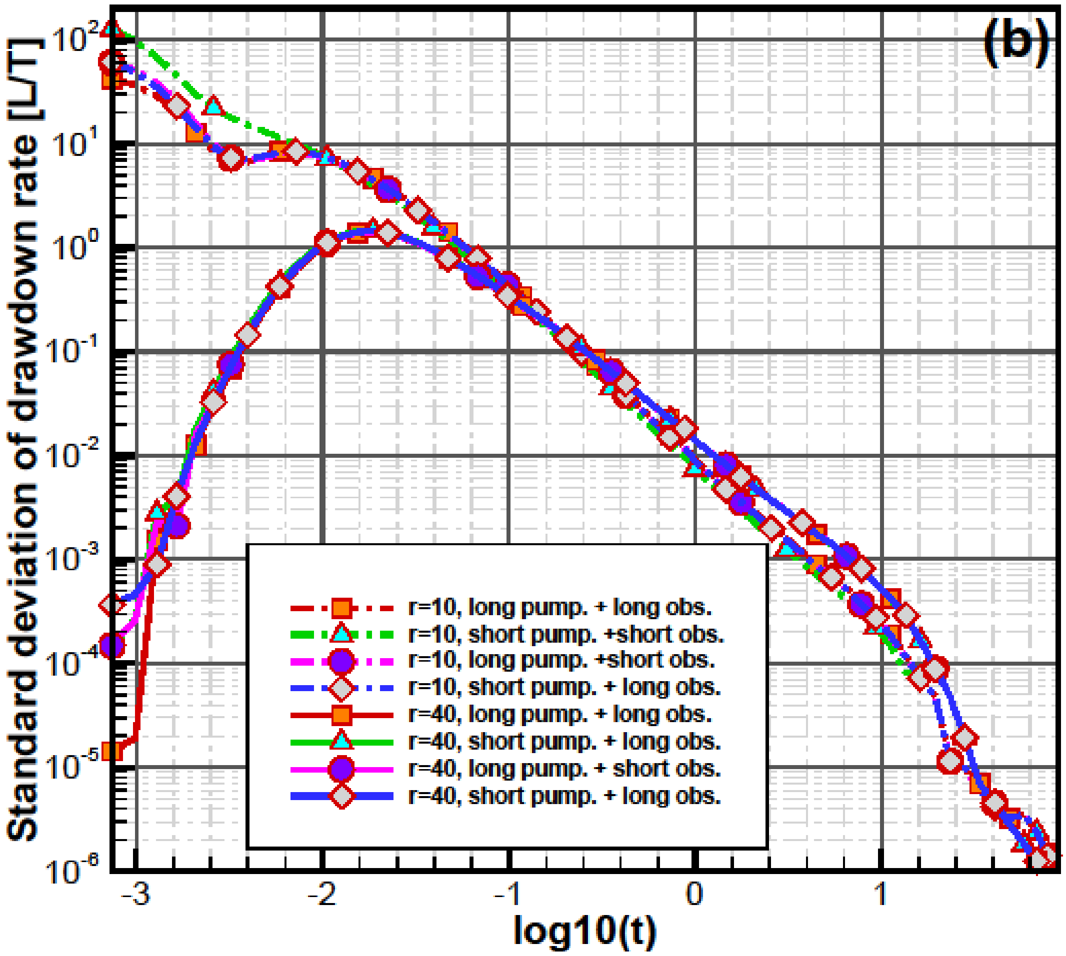
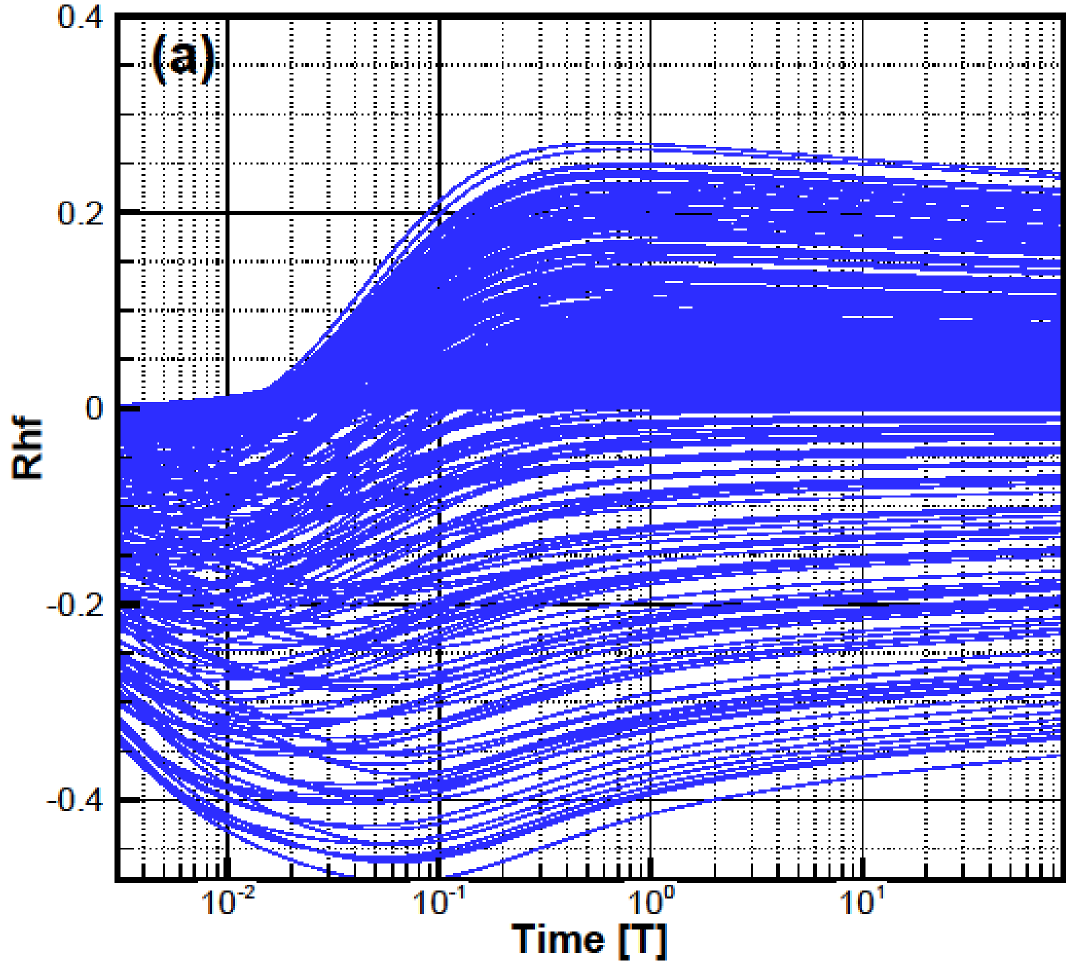


Fig. 9. The temporal evolution of correlations between hydraulic head $h(t)$ at the observation well and parameter (a) ln$K$ or (b) lnS_s everywhere versus time t. In both case, the variance is 1 and correlation scales are 10, 10, and 3.



(a)
0.4
0.2
0
-0.2
-0.4
Rhf
10^{-2}
10^{-1}
10^{0}
10^{1}
Time [T]

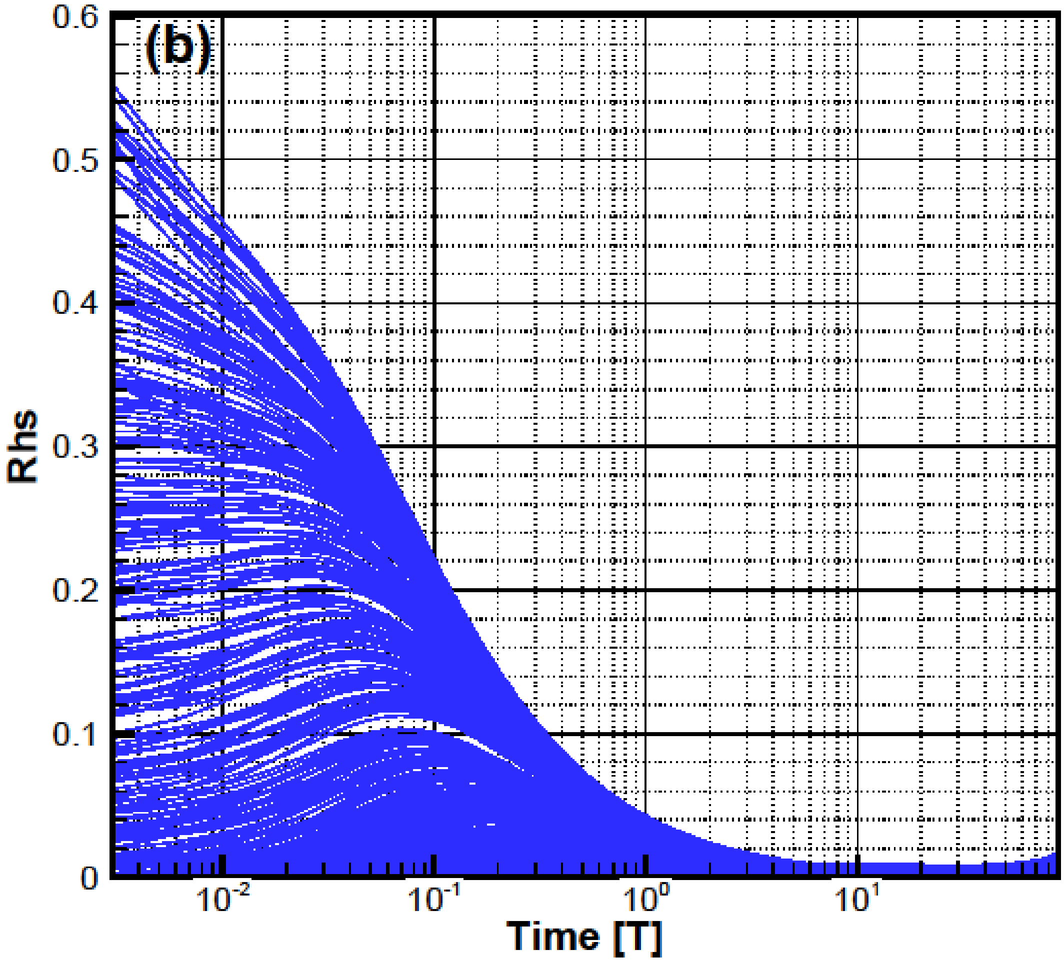


Fig. 10. The temporal evolutions of drawdown *s*, its derivative *w* and their bounds (adding or extracting one standard deviation) collected in nine pumping tests conducted at the NCRS site. Red solid line is the mean value calculated from 280 curves at different

observation points. Drawdown data are normalized by their pumping rates and the horizontal axis $t' = t/r^2$ is normalized by r^2.

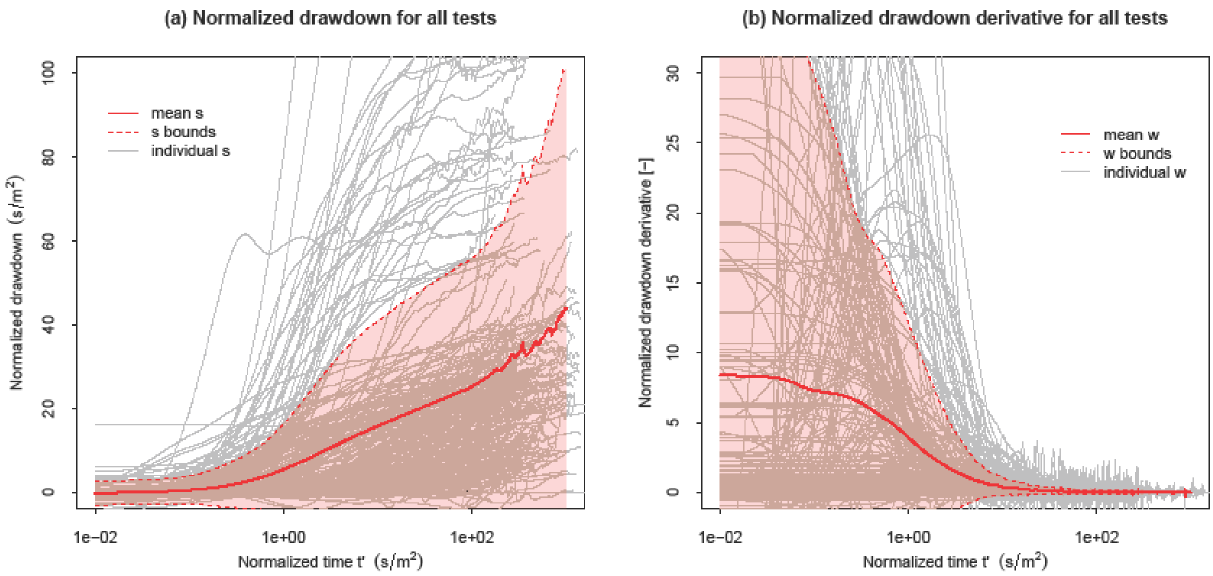


Fig. 11. The temporal evolutions of drawdown *s* and its derivative *w* during the pumping test conducted at PW1-3 at the NCRS site. The horizontal axis $t' = t/r^2$ is normalized by r^2.

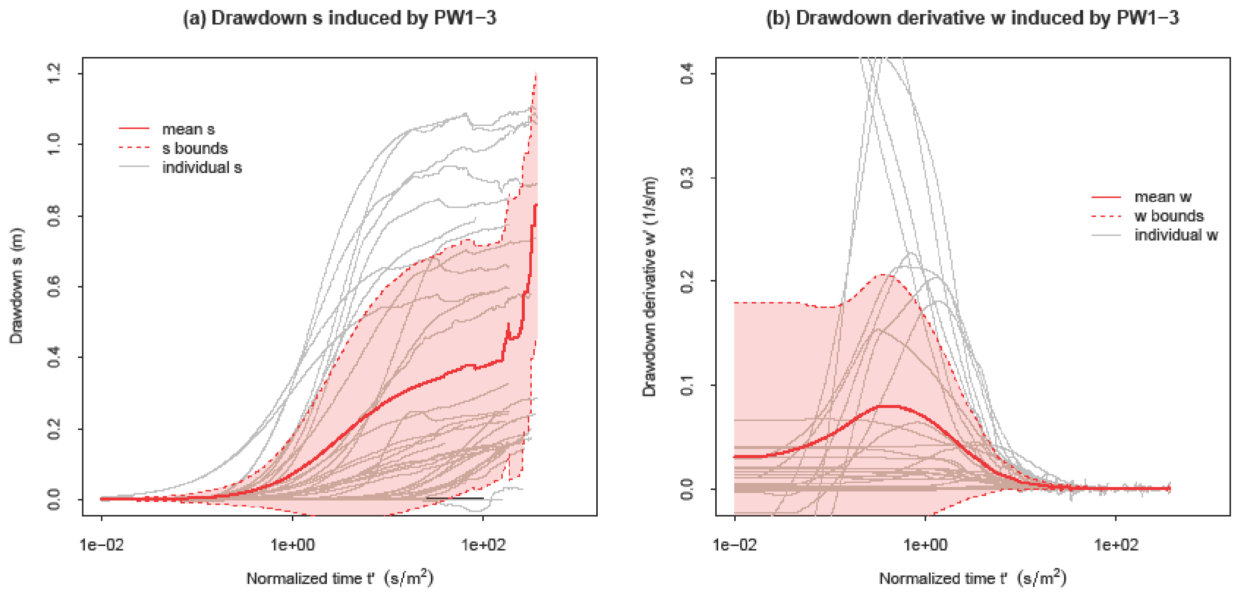
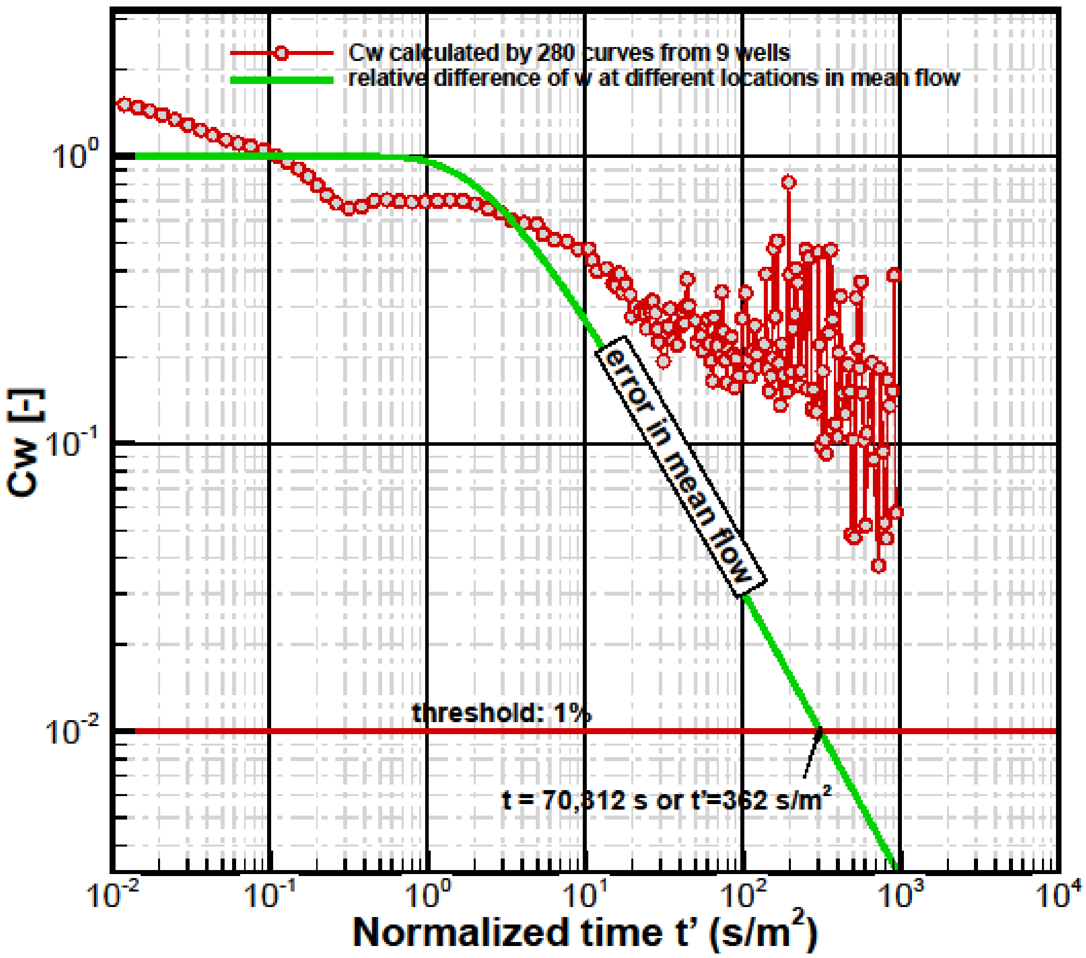


Fig. 12. Coefficient of variation C_w (defined by standard deviation of *w* normalized by *w*) versus time calculated by the 280 drawdown-time curves collected at NRCS site. The horizontal axis $t' = t/r^2$ is normalized by r^2. The green line is 1-exp(-*u*) indicating the relative error of *w* in the mean flow according to Eq. (3).


Cw calculated by 280 curves from 9 wells
relative difference of w at different locations in mean flow
error in mean flow
threshold: 1%
t = 70,312 s or t'=362 s/m²
Cw [-]
Normalized time t' (s/m²)

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